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Seismic Response of Stiffening Elastic Systems

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ABSTRACT

Seismic Response of Stiffening Elastic Systems

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Traditional seismic load resisting systems in buildings are designed to undergo inelastic deformations in order to dissipate energy, resulting in residual displacements. This work explores an approach to eliminate these residual displacements. The systems investigated have low initial stiffness which increases at a predefined displacement, and are therefore called stiffening elastic systems. This thesis begins with an examination of single-degree-of-freedom stiffening elastic systems. A case study is presented which suggests that the benefits from stiffening elastic behavior may be limited to systems which would have long periods if designed traditionally. A thorough parameter study is also presented which indicates the benefit of stiffening elastic behavior for SDOF systems with periods greater than four seconds. A final case study is presented that compares the response of a twelve-story stiffening elastic system to a ductile system and an elastic system. The stiffening elastic system was able to eliminate the residual displacements inherent in a ductile system while lowering the base shear experienced by the elastic system, but is not clearly better than the ductile system because the base shear force was much higher.

Keywords: earthquake engineering, response history analysis, performance based design

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1 INTRODUCTION

1.1 Seismic Design of Structures

Earthquakes and their associated effects present a significant challenge to structural engineers. Considerable time and resources have been spent over the years researching methods to mitigate the effects of earthquakes and reduce the damage a structure may experience under severe earthquake loads.

Since seismic design requirements began to appear in building codes in the 1920s, the primary focus has been the protection of life safety. Current practice is to design the seismic load resisting system to prevent damage and maintain operability during smaller earthquakes and to prevent collapse and protect life safety in the event of a large earthquake. However, outside of collapse prevention, there is no specified limitation for damage sustained by the structure during a large earthquake. Discovering ways to economically design structures to survive large earthquakes and remain operable with minimal repair is an important field of structural engineering research (Hamburger 2009).

1.2 Ductile Systems

Ductile systems are designed to respond inelastically to large earthquakes. Ductility is the ability to maintain strength even after the system yields and loses stiffness. This inelastic response dissipates the energy of the earthquake in the form of plastic deformation, but also leads to higher drifts during the earthquake and residual drifts after the earthquake. The amount of damage a structure experiences is largely determined by these drifts. Therefore, designing a structure to survive a large earthquake with minimal damage is not possible using ductile systems.

The common ductile seismic load resisting systems are steel braced frames and steel moment frames. Braced frames resist lateral forces through the axial stiffness of diagonal bracing much like a truss. Braced frames are typically more economical than moment frames because the members can be much smaller and the connections simpler. The most common type of braced frame is the concentrically braced frame, with braces which connect at the center of beams or at beam-column connections. Buckling-restrained braced frames are a special type of concentrically braced frame that allow the braces to yield in tension and compression, providing higher ductility.

Moment frames resist lateral forces through the flexural stiffness of beams and columns detailed such that the connections can transfer moments between the members. Because of this, they typically require heavier members and more labor-intense connections. Moment frames are considered advantageous because they do not require braces that interfere with architectural features and are considered more ductile than braced frames.

An advantage of braced frames over moment frames is that the braces are designed to act as a fuse, meaning that damage is limited to the braces and prevented in the beams and columns. The braces are more easily replaced than beams and columns. Damage in moment frames occurs in the beams and columns of the structure, affecting the stability of the structure and making repairs much more difficult.

1.3 Dampers

Ductile seismic load resisting systems are designed to dissipate seismic energy through inelastic deformation of specified members. Dampers provide an alternate method of dissipating this energy, which is why they are sometimes referred to as "energy dissipating devices." The primary purpose of dampers is to reduce the amount of energy dissipation required through the release of strain energy, thereby reducing the amount of inelastic deformation and damage to framing members (Constantinou and Symans 1993). Two common types of dampers are friction dampers and viscous dampers. Friction dampers are considered displacement-dependent and dissipate energy through heat from the friction between pads that displace relative to each other during an earthquake. Viscous dampers are considered rate-dependent and dissipate energy by forcing a fluid through orifices.

1.4 Seismic Isolation Systems

Seismic isolation systems are another method for seismic control of new structures and retrofitting existing structures. The purpose of seismic isolation is to increase the fundamental period of the structure, moving the system to the right on the response spectrum and reducing the design acceleration and seismic forces. Isolation systems are typically installed at the base of the structure which allows them to be good retrofit solutions.

1.5 Stiffening Elastic Systems

This thesis explores a different type of passive seismic control system with a unique hysteretic behavior. This system would be designed such that it would have little or no stiffness at low displacements and provide increasing stiffness as displacements increased. The theory being that the initially "soft" system would increase the period of a structure in the same manner as base isolation systems and might decrease the seismic load on the structure. The system would then stiffen when required and be able to elastically resist the reduced load. Because of this behavior, it is called a stiffening elastic system.

A stiffening elastic system could be created by a variety of methods, such as two different systems in series, using rubber constrained in a steel cylinder, or even with magnets. The purpose of this study is not to develop a system that exhibits this behavior, but to analytically investigate the dynamic response of such a system to determine if further investigation of stiffening elastic systems is warranted.

The stiffening elastic system would be a substitute for ductile systems in an effort to eliminate residual displacements. It would be more closely related to base isolation systems than dampers because its purpose is to shift the period of the structure and not to dissipate energy in a nondestructive manner like a damper. However, as will be shown in the following chapter, it borrows ideas and principles from all types of seismic load resisting systems.

1.6 Thesis Outline

The first chapter of this thesis establishes the background and motivation of the study. Chapter two presents a literature review of current practice and research pertinent to stiffening elastic systems. Chapter three describes a study of single degree-of-freedom stiffening elastic systems. Chapter four describes a study of multiple degree-of-freedom stiffening elastic systems. Chapter five summarizes the study and conclusions, including some implications. Supplemental information can be found in the appendices.

2 LITERATURE REVIEW

2.1 Overview

The stiffening elastic system being investigated would be designed as the primary lateral force resisting system of the structure and would exhibit properties of dampers, isolation systems, and self-centering systems. Several analytical, numerical, and experimental studies have been conducted on these and other methods of seismic control. This chapter provides an overview of that research which is pertinent to stiffening elastic systems.

2.2 Energy Dissipation Systems (Dampers)

Dampers may be divided into two major categories, passive dampers and active dampers, as described in the introduction. Stiffening elastic systems are inherently passive, but are able to take advantage of some of the benefits of active dampers. Research on each type of damper is described in the following sections.

2.2.1 Passive Dampers

Passive dampers are devices which use the relative displacement of the attachment points to produce control forces or dissipate energy. Symans et al. summarized the current practice and recent developments in the exploration of passive dampers (2008). Marshall and Charney conducted an investigation of a hybrid system which combined the two main types of passive dampers: rate-dependent and a rate-independent damper (2011). They performed an analytical study of several multi-story building models with ductile seismic load resisting systems, such as moment frames and buckling-restrained braced frames, and different configurations of the hybrid system. Some of these configurations resulted in a stiffening elastic system which increases in stiffness with displacement. These hybrid systems “were shown analytically to improve structural response to seismic events,” providing some hope that stiffening elastic systems could be advantageous.

2.2.2 Active Dampers

Active dampers can be more effective in reducing the seismic response of a structure than passive dampers because they employ feedback control systems which monitor the response of a structure during a seismic event and adjust the properties of the damper accordingly. This behavior allows the damper to adapt to the unique vibration of any event. However, these systems are complicated and demand power during operation, which may be interrupted or not available during an earthquake. To overcome these issues, semi-active dampers have been developed which equip passive dampers with actively controlled parameters.

One of the more promising recent developments in seismic control is the semi-active magnetorheological dampers. These dampers were demonstrated numerically by Dyke et al. (1996) to reduce the displacement of a three-story building model under a seismic load by about 30%. They later verified their results experimentally with a scaled down model of the building on a shake table (Dyke et al. 1998). Xu et al. performed a parametric investigation of magnetorheological dampers in a five-story building model (2000). They found that while the dampers themselves were an improvement over traditional seismic control methods, the improvement due to these dampers could be increased with the right parameters and control strategies.

2.3 Base Isolation Systems

Base isolation systems work best for low-rise, short-period structures. With taller structures, the increased period of the base isolation system may more closely match the fundamental period of the structure and result in resonance. The goal of stiffening elastic systems is to provide similar benefits to base isolation systems and also be applicable to tall, long-period structures.

Two approaches to achieve base isolation are to provide a layer of low lateral stiffness, typically with short, cylindrical bearings composed of hard rubber and steel, or to provide a system of sliding elements, such as a friction pendulum system (Chopra 2007). Both of these approaches are considered passive base isolation systems.

Abe et al. conducted an experimental study of three different types of rubber base isolation bearings (2004). They performed biaxial and triaxial tests on bearings comprised of high-damping rubber, natural rubber, and lead rubber. They found that the systems behaved as expected, increasing in stiffness with displacement and providing a restoring force to the structure, which is the type of behavior hoped for from the stiffening elastic system.

Passive base isolation systems may be equipped with control actuators to create an active base isolation system with improved performance over a wider range of earthquakes. Chang and Spencer conducted an experimental study of a system comprised of a scaled three story building with computer controlled actuators on a shake table (2010). They found the system reduced accelerations and base shear while limiting displacements over a wide range of seismic excitations. Bani-Hani and Sheban performed an analytical study of a five story structure with different types of base isolations systems (2006). They demonstrated that magnetorheological dampers applied to the base isolation system, with the right control strategy, are much more effective than ordinary base isolation systems. This indicates that an active stiffening elastic system may be even more effective at improving the response of a structure than its passive counterpart.

2.4 Self-Centering Systems

Ductile systems typically result in large residual displacements after a significant seismic event. Self-centering systems are attractive because they eliminate these residual displacements and return the structure to its original shape after an earthquake. Self-centering behavior can be achieved with mechanical devices or by using a special material, such as shape memory alloys, in a traditional system.

Shape memory alloys used in seismic applications were explored in a numerical investigation by Bruno and Valente (2002). These alloys have the ability to return to their pre-deformed state after stress is removed, but are not considered elastic because they do not unload on the same path that they are loaded. They have been used in applications ranging from couplings for oil pipelines to medical stents inserted into blood vessels to provide support. The investigation explored the alloys as a part of brace systems and base isolation systems in new and existing buildings and found that the primary benefit was the elimination of residual displacements, as the response of the buildings were not significantly improved.

Another self-centering method, the use of post-tensioned energy dissipating devices, was explored by Christopoulos et al. (2002). These devices exhibit a flag-shaped hysteretic behavior in both tension and compression. They modeled this behavior in a numerical analysis program and observed that the performance of the system was always at least as good as a comparable elastoplastic system with the added benefit of no residual displacement.

A similar system was investigated analytically by Tremblay et al. (2008). Buildings of different heights were designed with elements comprised of steel braces interconnected by a friction energy dissipating device and equipped with pretensioned steel cables providing the self-centering capability. The response of these buildings under a suite of earthquakes was compared to the same building designed using buckling-restrained braces. They found the self-centering devices preferable because of the elimination of residual displacements, even though the accelerations of the structure with these braces were slightly higher.

The primary advantage of the proposed stiffening elastic system, like self-centering systems, is the elimination of residual displacements. However, the stiffening elastic system would do so through ultra-flexibility, rather than relying on mechanical systems or material properties to pull the building back to its original shape.

2.5 Sequential Coupling

A final method of seismic control to be discussed is known as sequential coupling and was proposed by Weidlinger (1996). Rather than dissipating energy or adding resistance like other seismic protection strategies, this system takes advantage of a specific property of oscillatory response, namely that "the amplitude of any local maximum response may be reduced if the amplitude of the immediately preceding local minimum is increased by reducing the yield resistance." A sequential coupling system accomplishes this with a series of elastoplastic systems which creates a longer period but no reduction in strength.

To demonstrate the advantages of sequential coupling, Weidlinger conducted a numerical investigation of a sequential coupling system applied to a single-degree-of-freedom model. The model, which had properties typical of a medium rise building, was subjected to four acceleration time histories. The results of the analysis showed as much as a 20-50% reduction in the response. He concluded that these systems were effective in reducing the cost of the seismic load resisting system and reducing, or even eliminating, residual displacements. The objective of the present work is to show that stiffening elastic braces can provide the same advantage, but do so without the inelastic behavior found in the elastoplastic systems.

3 SINGLE DEGREE OF FREEDOM ANALYSIS

3.1 Overview

The first step of the investigation of the stiffening elastic system was to observe its behavior in a single-degree-of-freedom (SDOF) model. This simple model allowed the comparison of its behavior against ductile systems and over a wide range of parameters. This chapter describes the methods and results of the SDOF study.

3.2 Computer Program

The software used in this study was OpenSees, an open source program developed by researchers at the Pacific Earthquake Engineering Research Center (PEER) as a tool for earthquake analysis of structural and geotechnical systems. A variety of scripts were written in tool command language (tcl) to perform the analyses described in this thesis, the source code of which can be found in Appendix A. The syntax for particular elements of the software is given parenthetically in the following sections describing the analysis. Further details for each element can be found in the OpenSees user's manual (PEER 2012).

3.3 Modeling

The SDOF system for this analysis was in the form of the braced frame shown in Figure 3-1. The columns and beam of the frame were modeled as truss elements (Truss) while the brace was modeled as a corotational truss element (corotTruss), which is better suited to model members with damping because it accounts for strain rate effects. The beam and columns of the frame were assigned an elastic material (Elastic) with very large cross-sectional area to isolate significant axial deformations to the brace. The stiffness of the system was controlled by modifying the cross-sectional area of the brace. Damping in the system was applied using the Rayleigh model, which obtains a damping matrix from the mass matrix and stiffness matrix each multiplied by a given factor. For any SDOF system, either the mass matrix or stiffness matrix may be used alone. In this case, the stiffness matrix was multiplied by a factor to result in a damping value of 2%.

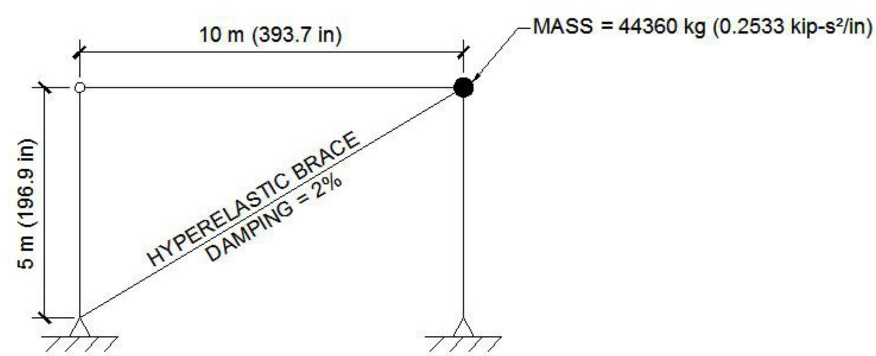

Figure 3-1: Frame model used in SDOF analysis.

The stiffening elastic behavior can be defined by three parameters: k, the final stiffness of the system; δ_y, the displacement at which the system increases stiffness; and α, the ratio of initial to final stiffness. These parameters are shown in Figure 3-2.

Because a stiffening elastic material is not available in OpenSees, it was necessary to create a material with the desired behavior. This was done by combining three materials which are found in the software. The first was a perfectly elastic material (Elastic) to provide the initial stiffness. The other two were perfectly plastic gap materials (ElasticPPGap), one with a tension gap (positive values for $gap and $Fy) and the other with a compression gap (negative values for $gap and $Fy) to provide the final stiffness. The gap materials were each assigned large yield strengths ($Fy) so that the system would remain elastic. The stiffening elastic material was created as a parallel material (Parallel), which, as the name implies, allows the three different materials to work in parallel as a single material. The stress-strain behaviors of the three individual materials are shown in Figure 3-3.

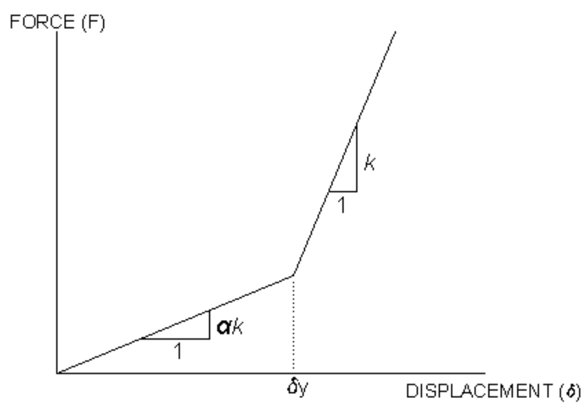

Figure 3-2: Parameters used to describe behavior of stiffening elastic system.

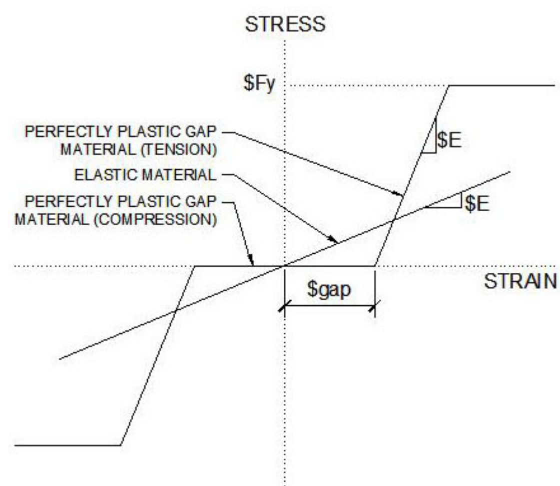

Figure 3-3: Three material types used in parallel to model the stiffening elastic system.

3.4 Case Studies

In order to see general advantages and disadvantages of the stiffening elastic system, the response of some systems subjected to the 1940 El Centro acceleration time history are presented in this subsection. Figure 3-4 shows the response and hysteretic behavior of two frames, one with a stiffening elastic brace and one with a ductile (elasto-plastic) steel brace. In order to compare the two frames, the final stiffness of the stiffening elastic frame was defined as the initial stiffness of the steel braced frame. The steel braced frame had a period of 1.0 second.

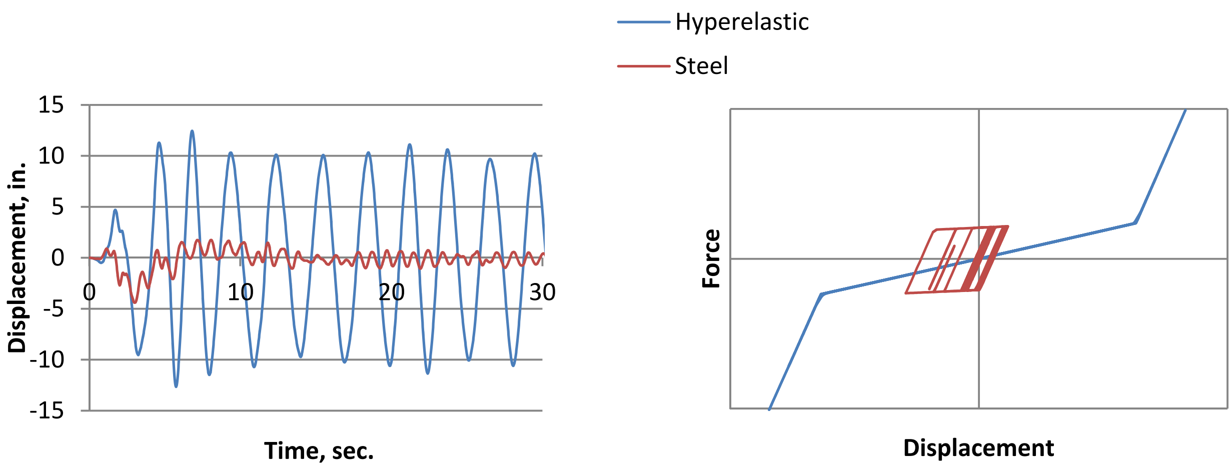

Figure 3-4: Displacement time history and hysteretic behavior of 1-sec period frames.

The stiffening elastic frame experienced 290% of the displacement and 440% of the base shear of the steel braced frame while eliminating only 3.5 inches of residual displacement. This is a small benefit relative to such a large increase in the response. For this case, the stiffening elastic system is clearly inferior to the ductile system.

Figure 3-5 shows the response and hysteretic behavior of another pair of frame. Again, the final stiffness of the stiffening elastic frame is the same as the initial stiffness of the steel braced frame, but in this case the steel braced frame had a period of 4.0 seconds.

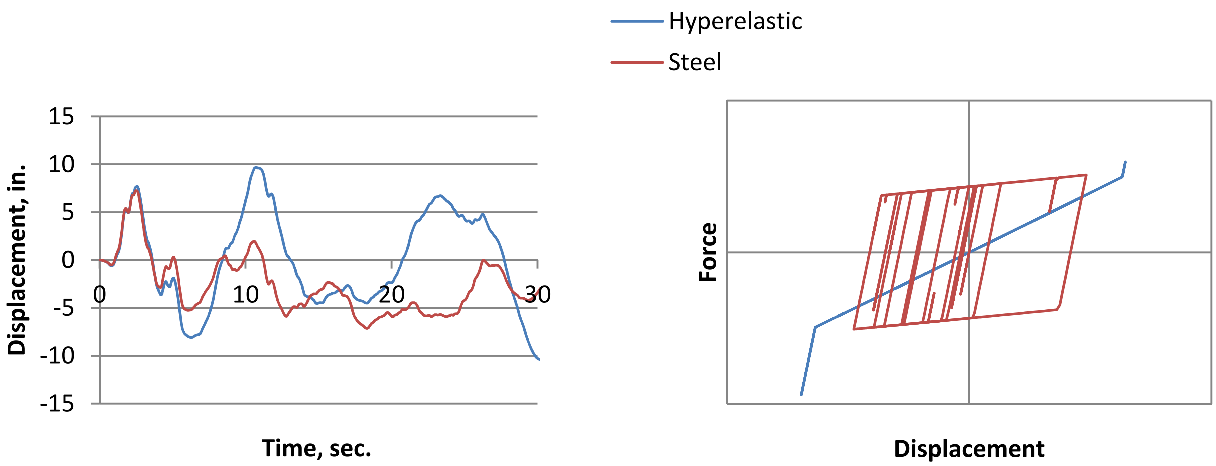

Figure 3-5: Displacement time history and hysteretic behavior of 4-sec period frames.

For this pair of frames, the stiffening elastic frame only experienced 140% of the displacement and 180% of the base shear of the steel braced frame while eliminating 6.2 inches of residual displacement. The significantly higher reduction in residual displacement could justify the increase in displacement and base shear.

These case study results suggest that the advantage of the stiffening elastic system may be for long period structures. A summary of relevant values from the plots is given in Table 3-1.

Table 3-1: Response Values From the Frame Analysis

	1-second Period		**4-second period**	
	Stiffening Elastic	**Steel**	**Stiffening Elastic**	**Steel**
Maximum Displacement, in.	12.7	4.4	10.4	7.3
Maximum Base Shear, kip	40.3	9.2	1.1	0.6
Residual Displacement, in.	0.0	3.5	0.0	6.2

3.5 Parametric Study

A parameter study of the SDOF system was conducted to determine the values for the three parameters of a stiffening elastic system that result in an improved structural response to seismic loads in terms of accelerations and displacements. The parameters of the same SDOF system as in the previous section were altered over a range of values and analyzed under a suite of ground motions. The analysis produced the relative displacement and absolute acceleration spectra for each case. The parameters examined in this study include the stiffness, k, the displacement where stiffness increases, δ_y, and the scaling factor, α.

3.5.1 Stiffening Elastic Parameters

Each spectrum produced by the analysis displayed the maximum response for systems with natural periods from zero to six seconds. The natural period of the system was modified by changing the stiffness of the brace, *k*, which was modified by changing its cross-sectional area.

The displacement at which the stiffness increases, $\delta_{y,}$ varied between 5 inches and 25 inches, which was close to the average maximum ground displacement of the suite of ground motions. While these displacements seem large for a single story frame, they are not unusual for taller, longer period structures. Because the stiffening elastic material was thought to be more effective for longer period structures, examination of larger displacements was important.

The ratio of initial to final stiffness, α, varied between 0.1 and 1.0, with 1.0 representing the elastic case.

3.5.2 Comparison Systems

The system was also analyzed with an elasto-plastic buckling-restrained steel brace. This brace was modeled with a simple steel material (Steel01) with a yield strength of 50 ksi, modulus of 29,000 ksi, and a post yield stiffness ratio of 0.02. The material performed the same in tension and compression like a buckling-restrained brace. This model allowed comparison of the performance between an elastic system, a ductile system, and various stiffening elastic systems.

3.5.3 Ground Motions

The acceleration time histories used for the parametric study were those developed for the SAC Joint Venture steel project (Somerville et al. 1997). These ground motions were selected and scaled to match the design spectrum for Los Angeles with a 10% chance of exceedance in 50 years over a wide range of periods. This made the suite ideal for the parametric study where it was important to determine at what periods the behavior of the stiffening elastic system outperformed the ductile system. Figure 3-6 shows the elastic spectra with 2% damping for each ground motion of the suite along with the mean response spectrum.

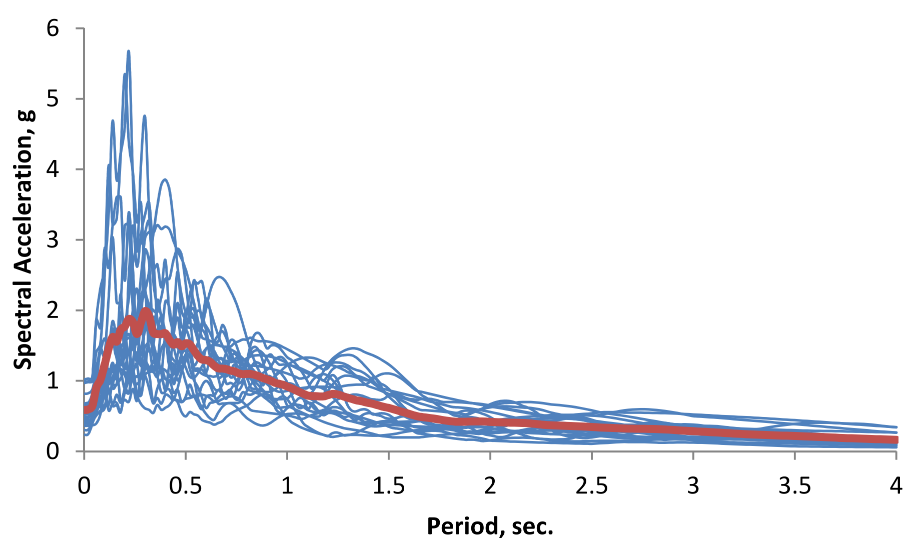

Figure 3-6: Elastic response spectra of ground motions used in the parametric study (2% damping).

3.5.4 Results of the Parametric Study

The results of the parametric study give further insight as to where the stiffening elastic systems may be considered advantageous. Figures 3-7 and 3-8 show the average response spectra of the different ground motions for the acceleration and displacement at the top of the frame for δ_y equal to 10 inches. The periods shown on these plots were calculated using the final stiffness of the stiffening elastic systems and the initial stiffness of the ductile systems, similar to what was done in the frame analysis above.

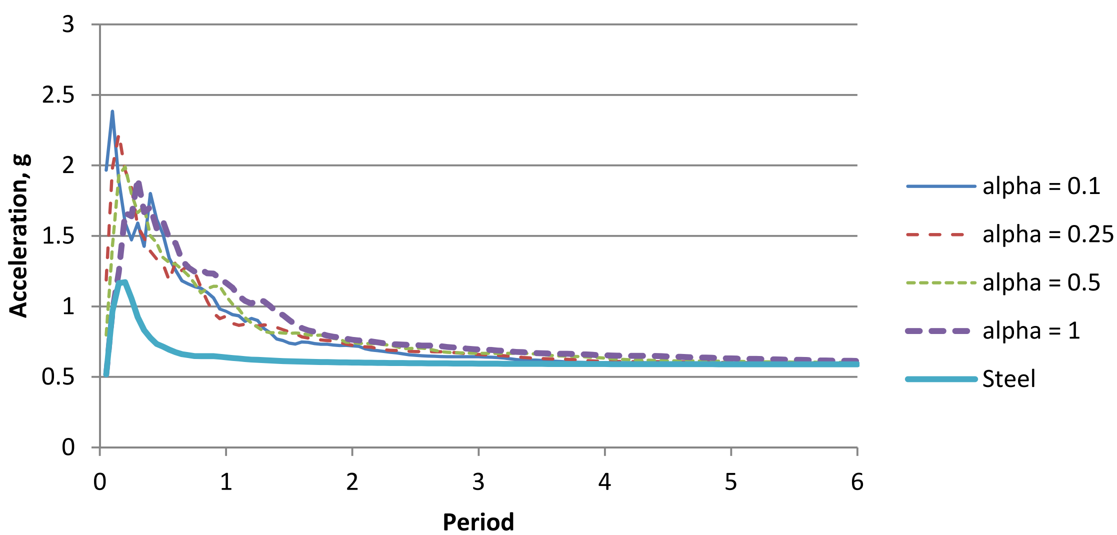

Figure 3-7: Average acceleration spectra for δ_y = 10.

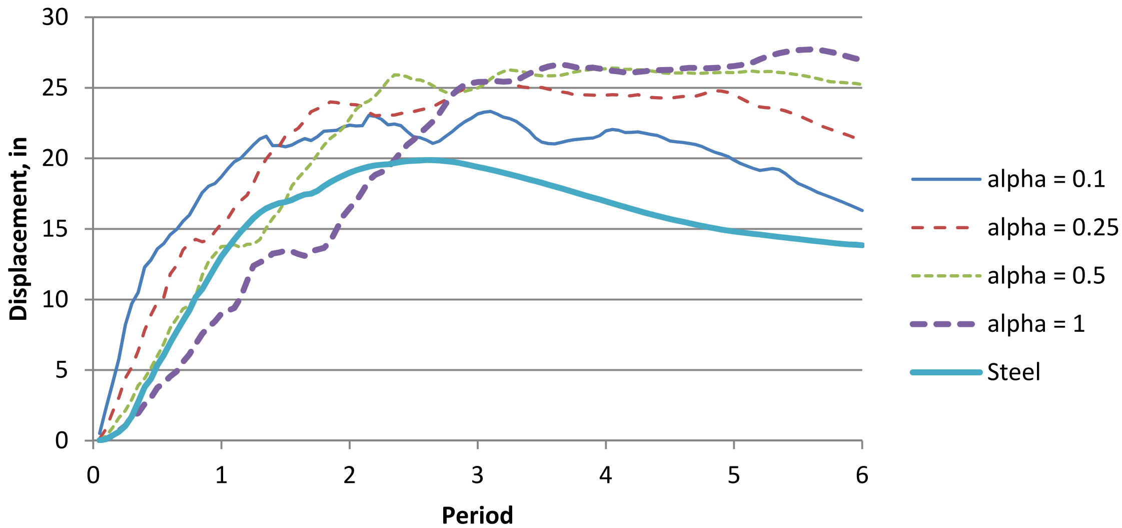

Figure 3-8: Average displacement spectra for $\delta_y = 10$.

The ductile system (steel) experienced less acceleration than all the other systems and less displacement than the stiffening elastic systems over the entire range of periods. The elastic system (alpha=1) experienced less displacement for shorter periods, but still had higher accelerations. This means that the forces in the system are higher, and constructing an elastic system to resist those forces would be cost-prohibitive.

Figures 3-9 and 3-10 show the average response spectra for δ_y equal to 25 inches and indicate that for natural periods greater than three seconds, the acceleration experienced by the different systems is nearly identical. For periods approaching four seconds, the displacement experienced by the stiffening elastic system with α equal to 0.1 approaches that of the ductile system. Meaning that for periods greater than four seconds, the accelerations and displacements of a stiffening elastic system and a ductile system are nearly the same. Plots of the results for the entire range of parameters investigated are presented in Appendix B.

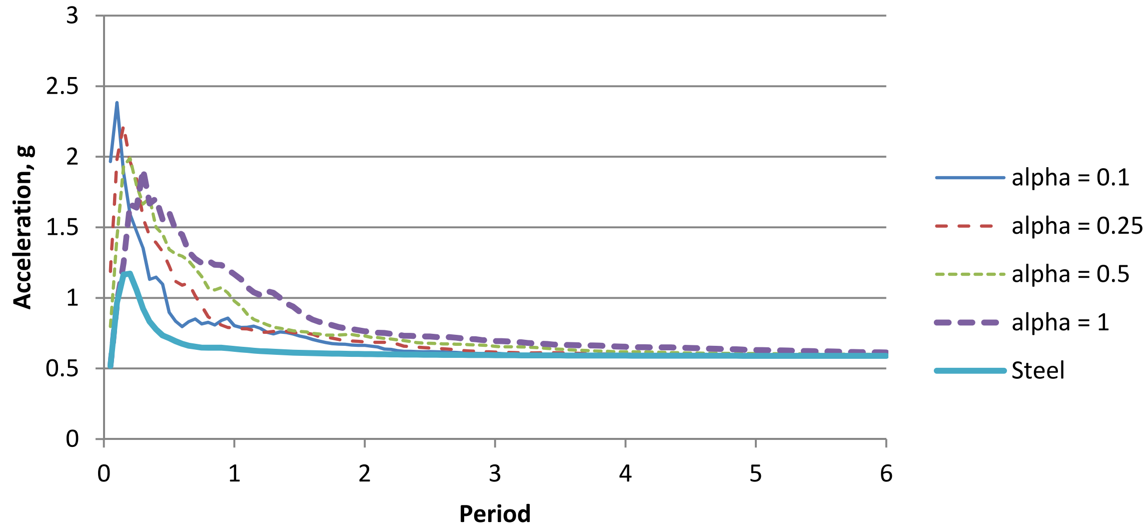

Figure 3-9: Average acceleration spectra for $\delta_y = 25$.

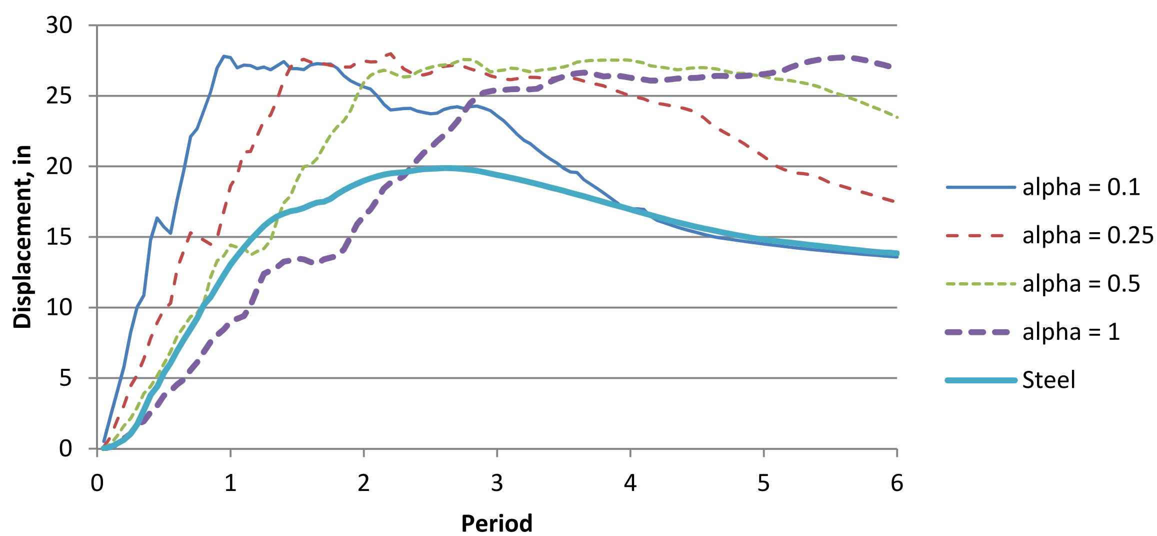

Figure 3-10: Average displacement spectra for $\delta_y = 25$.

The observation that displacements are the same for ductile systems and stiffening elastic systems with α equal to 0.1 is significant because the hysteretic behavior of the steel brace is such that there will be large amounts of residual displacements after a seismic event, while the stiffening elastic brace is designed to eliminate these displacements. Therefore, a stiffening elastic system with the right parameters could exhibit the advantage of eliminating permanent structural damage while experiencing no additional displacement or acceleration for long period structures. The following chapter will examine the application of a stiffening elastic system to a tall building to determine if these findings extend to multiple-degree-of-freedom systems.

4 MULTIPLE DEGREE OF FREEDOM CASE STUDY

4.1 Overview

While the results from a single-degree-of-freedom (SDOF) analysis can be useful in seeing general patterns in the behavior of stiffening elastic systems, most structures are not SDOF systems. Therefore, to understand the benefits of stiffening elastic systems in more realistic applications, a multiple-degree-of-freedom (MDOF) analysis is required.

This chapter outlines an MDOF case study which compared the response of a 12-story building equipped with a ductile seismic load resisting system and the response of the same building equipped with stiffening elastic braces. The ductile system in this case was comprised of buckling-restrained braced frames (BRBF). The analysis was performed with the same software used in the SDOF analysis. The responses measured were the acceleration and displacement at the roof of the structures, the story drift at each level, and the base shear.

4.2 Design of the BRBF Lateral Force Resisting System

The purpose of this study was to evaluate possible benefits of using stiffening elastic systems over ductile systems. Therefore, the simple structure shown in Figure 4-1 was designed with all gravity-load carrying elements designed to meet current codes. The calculations for this design are presented in Appendix C. The BRBFs were designed to resist the seismic loads prescribed by the equivalent lateral force method as described in ASCE-07 (ASCE 2010), assuming a seismic weight of 4 kPa (83.5 psf) at each level. The cross-sectional area of the braces at each story are shown in Table 4-1. The columns in the BRBFs were then designed based on the capacity of the braces.

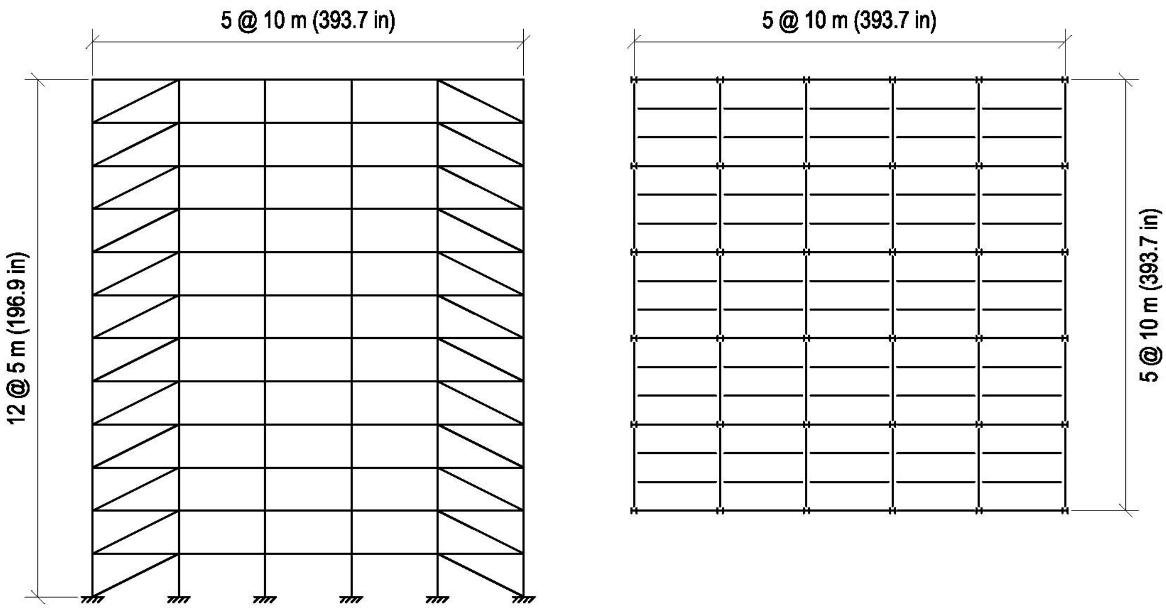

Figure 4-1: Elevation and plan of the structure used in the MDOF case study.

Table 4-1: Cross-Sectional Area of BRB at Each Story

Story	BRB Area, in^2
1	11
2	11
3	11
4	11
5	11
6	10
7	10
8	9
9	8
10	7
11	5
12	3

4.3 Design of the Stiffening Elastic Lateral Force Resisting System

To design the stiffening elastic lateral force resisting system (LRFS) for the structure, values of k, δ_y, and α needed to be determined for each level. For the LFRS to be practically applicable, it must allow the structure to stay within drift limits under wind loads. Wind loads were determined using ASCE 7 and a recommended story drift limit of L/400 was used to prevent non-structural damage. The minimum stiffness required at each level was determined to meet these limits.

In the SDOF study, it was determined that a value of 0.1 for α produced the best results, and therefore α was equal to 0.1 for each story in the MDOF study. The final stiffness, k, at each level was determined by dividing the minimum stiffness by α.

A value of 25 inches for δ_y was also selected from the SDOF study as a desirable value. In order for the system as a whole to increase stiffness at this displacement, values for δ_y at each level needed to be determined. This was done using a pushover analysis of the structure with the same loads determined above for the BRBF model and the parameters already decided on as noted above. Using guess and check, a δ_y value of 1.67 inches for each floor was determined to provide the sought after behavior.

The parameters in this section describe the baseline stiffening elastic system used to compare with the elastic and BRBF systems. The changes to the stiffening elastic system in later sections are all modifications to this baseline system. A summary of these parameters is given in Table 4-2.

Table 4-2: Parameters of the Stiffening Elastic System Case Study

Story	k, kip/in	α	δ_y, in
1	564.1	0.1	1.67
2	528.7	0.1	1.67
3	487.9	0.1	1.67
4	445.0	0.1	1.67
5	399.0	0.1	1.67
6	350.3	0.1	1.67
7	300.7	0.1	1.67
8	249.2	0.1	1.67
9	195.8	0.1	1.67
10	141.1	0.1	1.67
11	84.9	0.1	1.67
12	28.7	0.1	1.67

4.4 MDOF Analysis

The 12-story frame was analyzed under the same 20 ground motions used in the SDOF analysis. The output from the analysis included the maximum displacement and acceleration at the roof of the structure, the maximum drift for each story, and the residual drift for each story. The residual drift was determined by allowing the analysis to run long after the ground motions were finished.

4.5 Results of the MDOF Study

Figure 4-2 and Table 4-3 show the results of the MDOF analysis for three buildings. One is the building with BRBFs (see Table 4-1). Another is the same building with elastic braces. The final is the building with stiffening elastic braces (see Table 4-2).

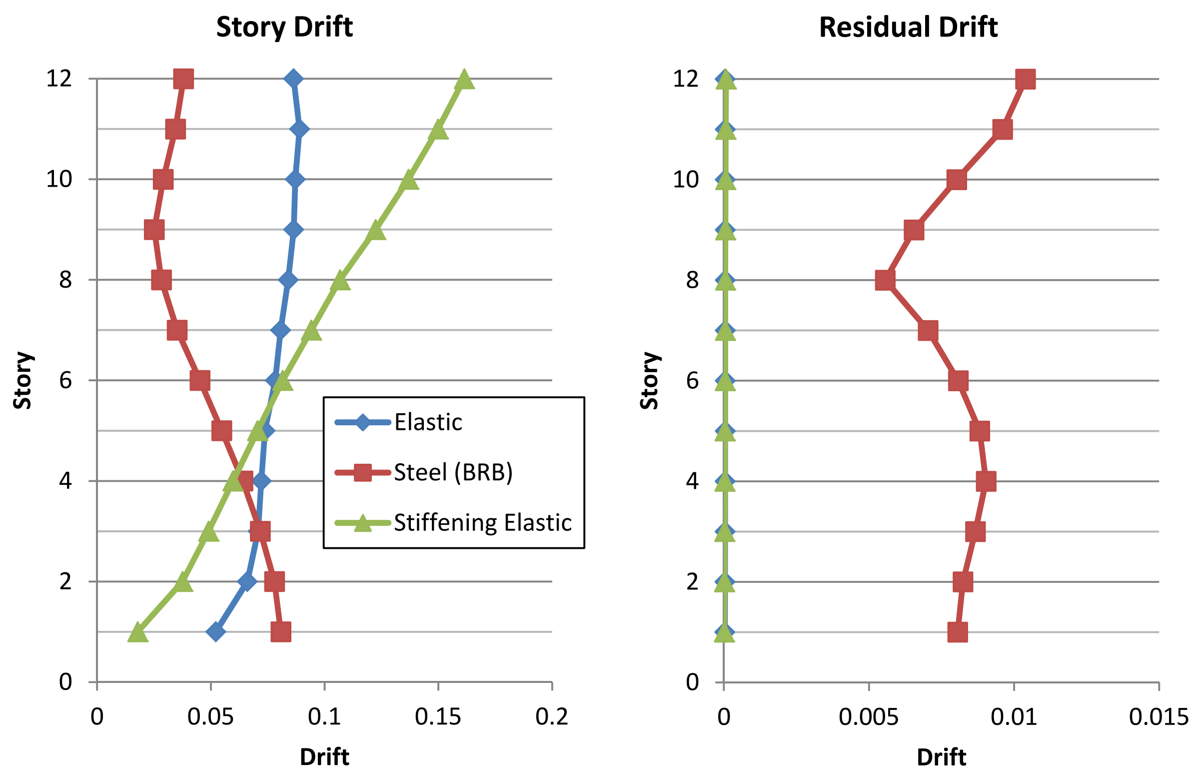

Figure 4-2: Story drift and residual drift of the three structures in the case study.

Table 4-3: Values from the Three Structures in the Case Study

	Displacement, in.	Acceleration, g	Base Shear, kip	Period, sec.
Elastic	172.46	2.41	5647	3.44
Steel	90.07	1.43	712	3.44
Stiffening Elastic	205.81	2.60	3323	5.65

From the SDOF study, it was expected that the stiffening elastic system with a long period would have comparable drifts to the BRBF system with the added benefit of the eliminating residual drifts. The MDOF study shows that the stiffening elastic system had almost five times the base shear and over twice the maximum displacement of the BRBF system (see Table 4-3). The stiffening elastic system also experienced story drifts of around 16% at the top of the structure. The elimination of residual drifts in this case does not justify such disadvantages.

The stiffening elastic system did exhibit an advantage over the elastic system, the other system that would not have residual drifts, in that the base shear was reduced by over 40% (see Table 4-3). This shows that it is possible to have a system that does not yield but may be more economical than a purely elastic system. However, the maximum displacements and story drifts are still more than what would be acceptable.

4.6 Discussion of Results

It is unclear why the stiffening elastic MDOF system did not perform as well as the steel MDOF system when the SDOF study suggested that they might be similar. One explanation might be that the interaction of multiple modes of vibration is very complex and hard to predict. The first mode of vibration of the MDOF system may behave much the same as the SDOF system, but higher modes cause the structure to behave quite differently. Vibration of an SDOF may be compared to shaking a stick while vibration of an MDOF system could be more like cracking a whip.

What can be done to account for this difference and have a multi-story structure with the benefits of a stiffening elastic system? The following sections describe some possible answers.

4.6.1 Increasing Story Stiffness

From Figure 4-2 above, it can be seen that the highest story drifts occur in the upper stories. Therefore, the first attempt to improve the performance of the multi-story frame was to increase the stiffness of each story by a factor, β. Because greater displacements were experienced near the top of the structure, it was desired that β would increase linearly from the bottom to the top of the structure. For example, if the stiffness of the top story was to be twice that of the original structure, β would be one for the bottom story, two for the top story, and be increasing linearly from one to two for the intermediate stories. Values of two through five for the stiffness increase at the top story were investigated, and the results for these systems are shown in Table 4-4.

Table 4-4: Values of the Stiffening Elastic Systems with Varying β

	Displacement, in.	Acceleration, g	Base Shear, kip	Period, sec.
Elastic	172.46	2.41	5647	3.44
Steel	90.07	1.43	712	3.44
Stiffening Elastic (β = 1 to 2)	212.90	2.81	4806	4.65
Stiffening Elastic (β = 1 to 3)	181.07	3.05	6071	3.95
Stiffening Elastic (β = 1 to 4)	155.27	2.99	6933	3.53
Stiffening Elastic (β = 1 to 5)	140.24	3.94	7286	3.23

From these results it can be seen that it is possible to decrease the maximum displacement of the system but it comes at the cost of increasing the base shear. The next step was to determine if changing β nonlinearly would produce better results. The factors used are shown in Table 4-5 and the results for this model are shown in Figure 4-3 and Table 4-6.

Table 4-5: Values of β for Each Story

Story	β
1	1.00
2	1.00
3	1.00
4	1.19
5	1.41
6	1.63
7	1.88
8	2.14
9	2.45
10	2.74
11	3.00
12	3.23

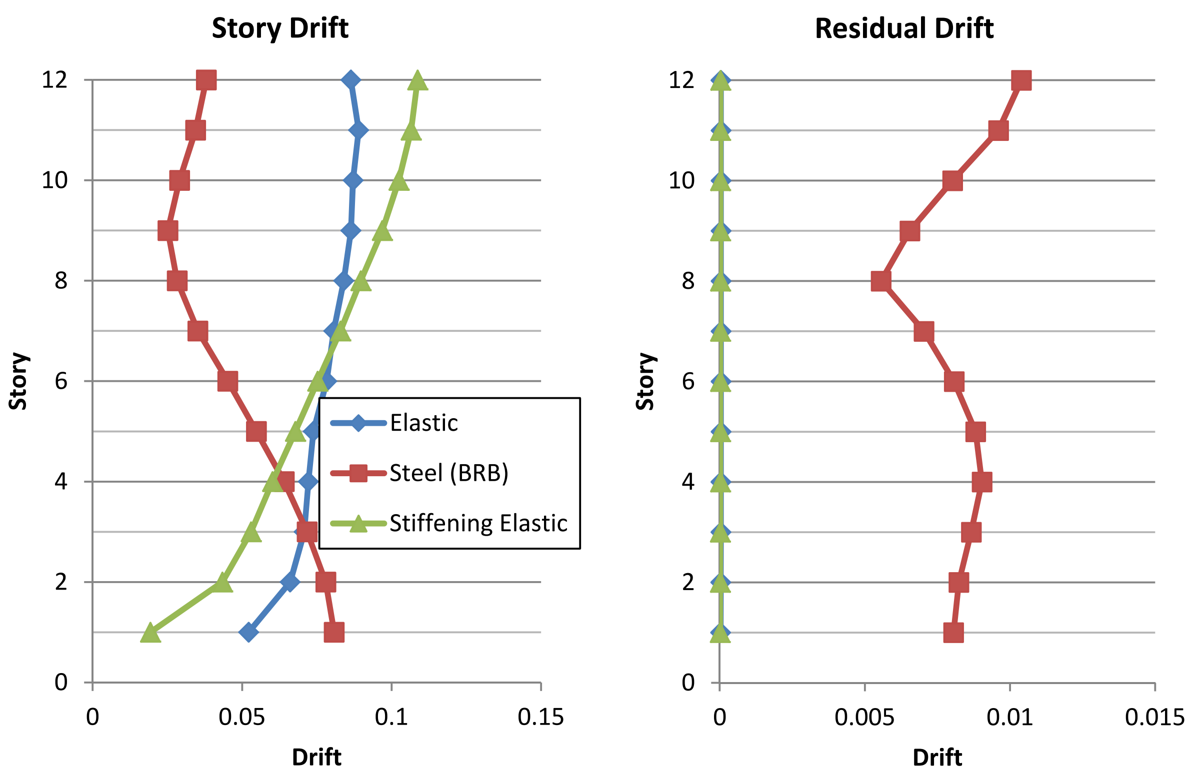

Figure 4-3: Story drift and residual drift of the model with nonlinearly varying β.

Table 4-6: Values for the Model with Nonlinearly Varying β

	Displacement, in.	Acceleration, g	Base Shear, kip	Period, sec.
Elastic	172.46	2.41	5647	3.44
Steel	90.07	1.43	712	3.44
Stiffening Elastic (β = 1 to 2)	212.90	2.81	4806	4.65
Stiffening Elastic (β = 1 to 3)	181.07	3.05	6071	3.95
Stiffening Elastic (β = 1 to 4)	155.27	2.99	6933	3.53
Stiffening Elastic (β = 1 to 5)	140.24	3.94	7286	3.23
Stiffening Elastic (Nonlinear β)	174.14	3.30	6279	3.81

Varying the value of β nonlinearly did not improve the performance of the system over any of the other stiffening elastic systems. The most improvement was found in the system where β varied linearly from one to two, but even this system performed worse than the baseline stiffening elastic system.

4.6.2 Mixing Stiffening Elastic and BRBF Systems

Another possible way to improve the performance of the multi-story structure would be to provide stiffening elastic systems at some floors and BRBF systems at other floors. Figure 4-4 shows the results for a system with a stiffening elastic system on every other floor and a BRBF system on the remaining floors. Figures 4-5, 4-6, and 4-7 show the results for a model with stiffening elastic systems on all but the top six floors, three floors, and one floor, respectively. Table 4-7 shows a summary of all these models.

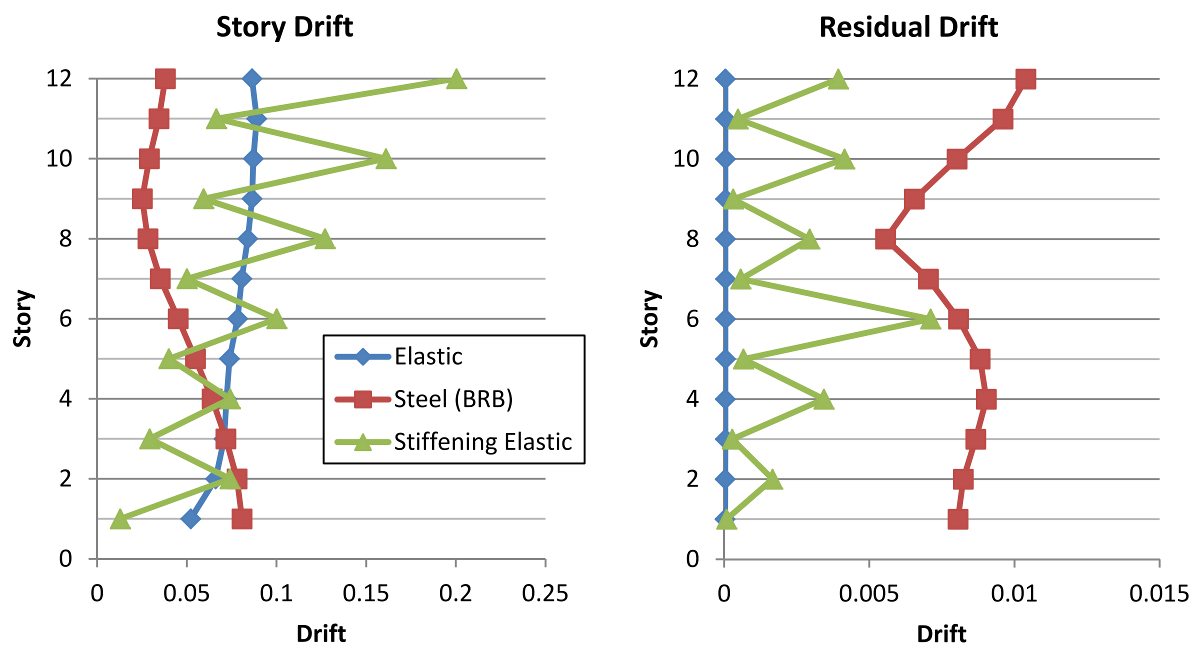

Figure 4-4: Story drift and residual drift of the model with alternating stiffening elastic and BRBF systems.

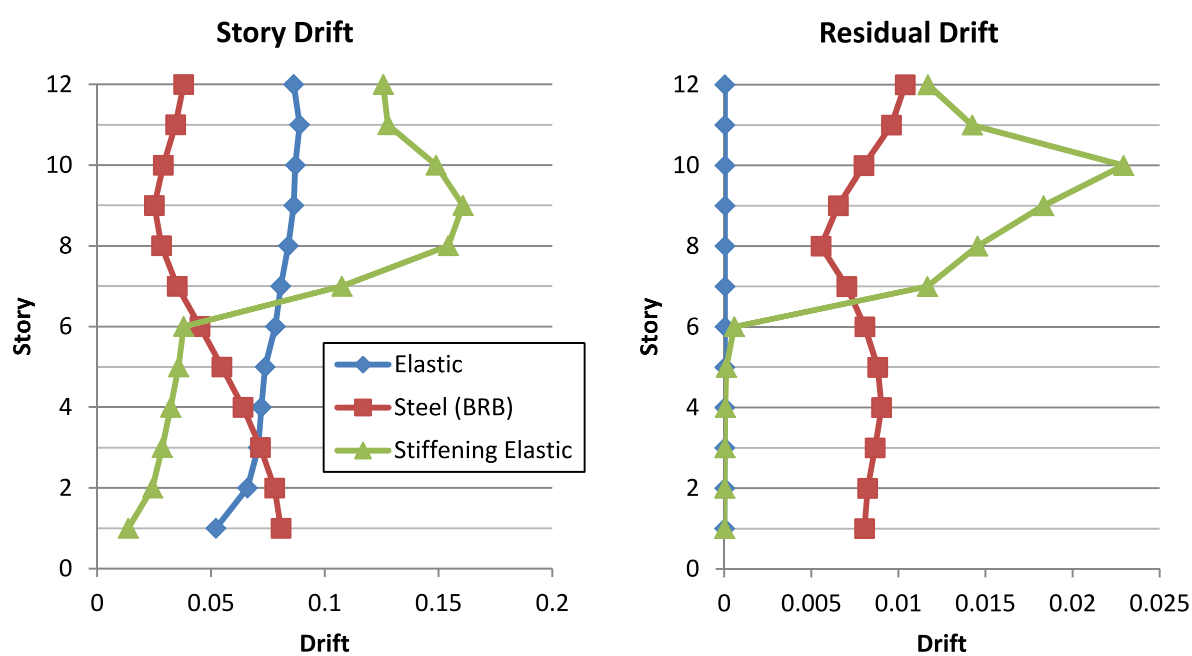

Figure 4-5: Story drift and residual drift of the model with stiffening elastic systems on the lower half.

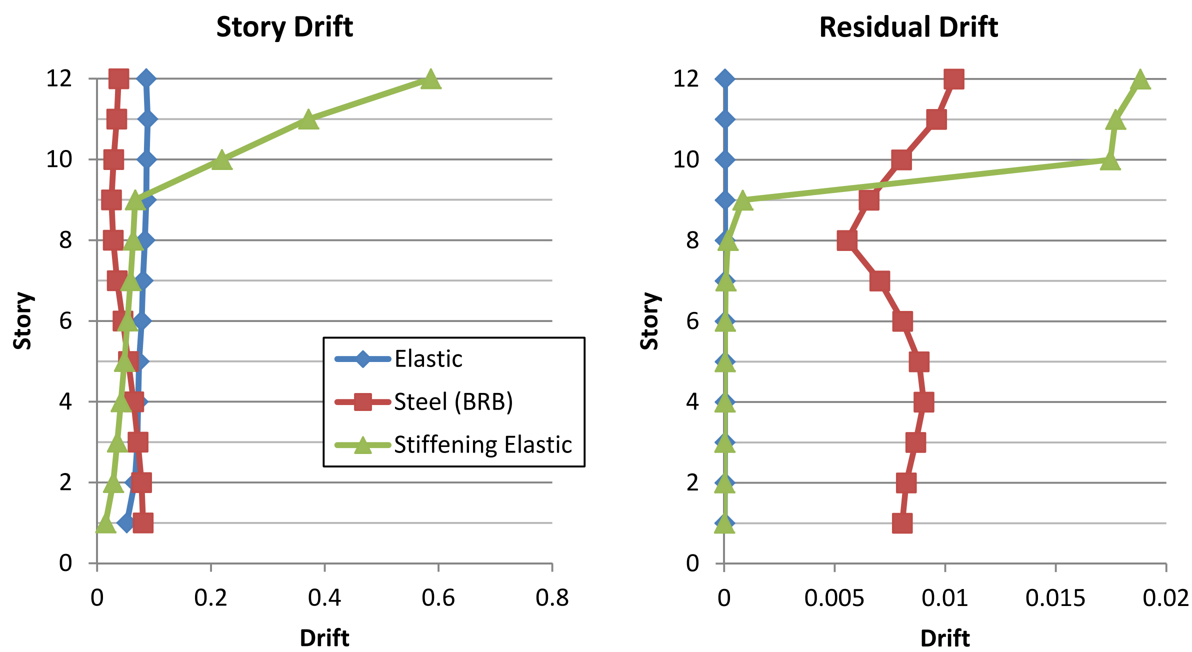

Figure 4-6: Story drift and residual drift of the model with stiffening elastic systems on the lower nine floors.

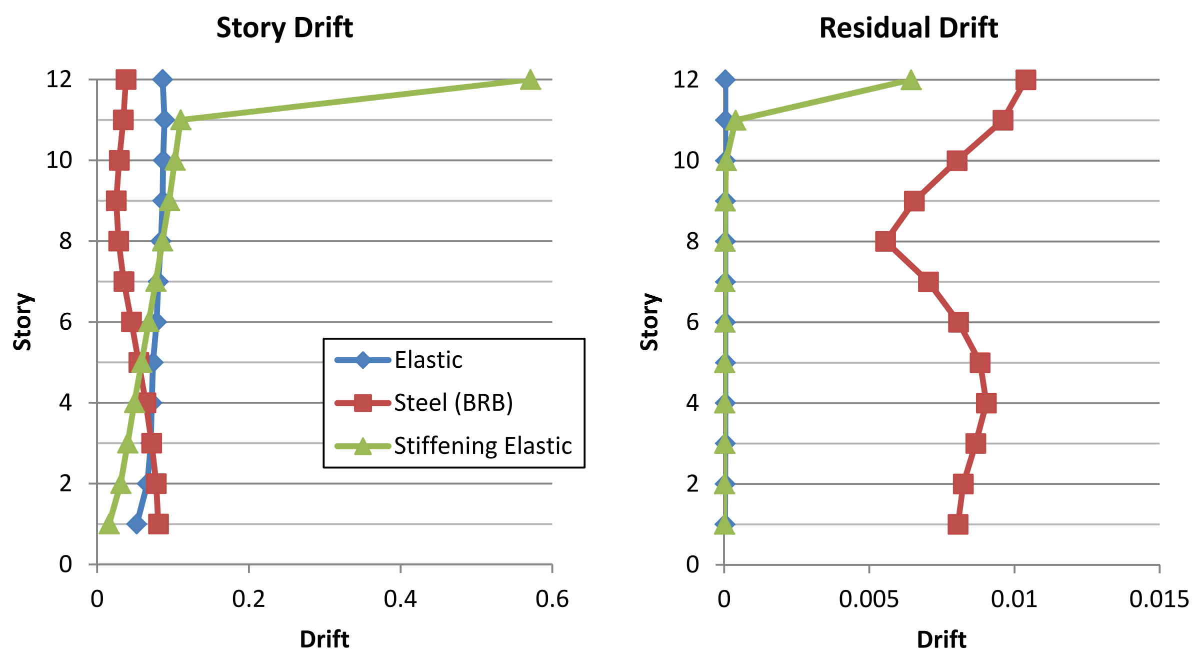

Figure 4-7: Story drift and residual drift of the model with stiffening elastic systems on all but the top floor.

Table 4-7: Summary of Results for Mixed Stiffening Elastic and BRBF Systems

	Displacement, in.	Acceleration, g	Base Shear, kip	Period, sec.
Alternating floors	172.36	1.44	1807	5.45
Bottom six floors	138.44	1.45	2488	5.39
Bottom nine floors	226.46	1.60	2484	5.52
Bottom 11 floors	221.88	2.03	2630	5.60

These results demonstrate that the floors with stiffening elastic systems performed better than the baseline structure when mixed with BRBF systems. However, the excessive drifts in the floors with BRBF systems make these kinds of mixed systems unreasonable.

5 CONCLUSIONS

The purpose of this research was to explore the behavior of a stiffening elastic system as the seismic load resisting system of a structure. The expectation was this system could lower the force imparted to a structure during an earthquake to a point where an elastic system would be practical. This would eliminate residual displacements in the structure and allow it to be fully functional after an earthquake.

The investigation began with an examination of the stiffening elastic system in a single-degree-of-freedom model. This demonstrated that the stiffening elastic system could be beneficial at very particular sets of parameters.

The next step of the investigation was to examine the stiffening elastic system in multi-story frame. A 12-story structure was used as the model and a buckling-restrained-brace system was designed to compare with the stiffening elastic system. The stiffening elastic system was designed and both systems were analyzed using OpenSees.

The stiffening elastic system was compared to an elastic system and a ductile system because they represent two extremes on the cost spectrum. Elastic systems have high base shears and therefore the overall cost of the structure is greater, but they experience no damage and therefore there is no repair cost after an earthquake. On the other end, ductile systems have low base shears and are cheaper to construct, but require extensive repairs or even replacement after an earthquake. The stiffening elastic system showed promise in the case study because it was able to reduce the base shear compared to the elastic system and eliminate the repairs required of an elastic system. Whether or not a stiffening elastic system would be desirable over a ductile system for a particular structure would depend on the magnitude of the savings in repair cost compared to the increase in construction cost.

This thesis has determined that stiffening elastic systems provide a promising avenue to improving the performance of structures under earthquakes and that further analytical research into these systems is warranted.

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APPENDIX A: OPENSEES SOURCE CODE

Found in this appendix is the source code for the single-degree-of-freedom and multiple-degree-of-freedom analyses.

```
#   -----FRAME CHECK-----

wipe all

set M 0.2533
set zeta 0.02
set pi 3.14159
set alpha 0.1
set deltaY 8.5

set T 1.0
puts "Analyzing Short Period (T = $T) Frames..."

for {set i 1} {$i <= 2} {incr i} {

puts "Analyzing frame $i..."

set Mat [expr $i*10]
set K [expr $M*pow(2*$pi/$T,2)]

source 1StoryFrame.tcl

wipeAnalysis
constraints Plain
numberer Plain
system BandGeneral
test NormDispIncr 1.0e-8 10
algorithm Newton
integrator Newmark 0.5 0.25
analysis Transient

set acceldata ElCentroChopra.at2
set dt 0.02
set nPts 1559

set accelSeries "Series -dt $dt -filePath $acceldata -factor 386.4"
pattern UniformExcitation 1 1 -accel $accelSeries
rayleigh 0. 0. 0. [expr 2*$zeta/(sqrt($K/$M))]

recorder Node -file "Data/ShortDisp$i.out" -time -node $myNode -dof 1 disp
recorder Node -file "Data/ShortForce$i.out" -node 1 -dof 1 reaction

analyze $nPts $dt

remove recorders

}
```

```
set T 4.0
puts "Analyzing Long Period (T = $T) Frames..."

for {set i 1} {$i <= 2} {incr i} {

puts "Analyzing frame $i..."

set Mat [expr $i*10]
set K [expr $M*pow(2*$pi/$T,2)]

source 1StoryFrame.tcl

wipeAnalysis
constraints Plain
numberer Plain
system BandGeneral
test NormDispIncr 1.0e-8 10
algorithm Newton
integrator Newmark 0.5 0.25
analysis Transient

set acceldata ElCentroChopra.at2
set dt 0.02
set nPts 1559

set accelSeries "Series -dt $dt -filePath $acceldata -factor 386.4"
pattern UniformExcitation 1 1 -accel $accelSeries
rayleigh 0. 0. 0. [expr 2*$zeta/(sqrt($K/$M))]

recorder Node -file "Data/LongDisp$i.out" -time -node $myNode -dof 1 disp
recorder Node -file "Data/LongForce$i.out" -node 1 -dof 1 reaction

analyze $nPts $dt

remove recorders

}

puts "Done"
```

```
#   -----FRAME SPECTRA-----

wipe all

set M 0.2533
set zeta 0.02
set pi 3.14159
set gamma 10.
set deltaY 9.0
set TStep 0.1
set TMax 10.0

set outFile "Data/Spectra.out"
file delete $outFile

for {set i 1} {$i <= [expr $TMax/$TStep]} {incr i} {

    set deltaY [expr $i*$TStep]
    set T 3.0

    for {set j 1} {$j <= 2} {incr j} {

        set Mat [expr $j*10]
        set K [expr $M*pow(2*$pi/$T,2)]

        source 1StoryFrame.tcl

        wipeAnalysis
        constraints Plain
        numberer Plain
        system BandGeneral
        test NormDispIncr 1.0e-8 10
        algorithm Newton
        integrator Newmark 0.5 0.25
        analysis Transient

        set acceldata ElCentroChopra.at2
        set dt 0.02
        set nPts 1559

        set accelSeries "Series -dt $dt -filePath $acceldata -factor 386.4"
        pattern UniformExcitation 1 1 -accel $accelSeries
        rayleigh 0. 0. 0. [expr 2*$zeta/(sqrt($K/$M))]

        recorder EnvelopeNode -file "Data/MaxDisp$j.out" -node $myNode -dof 1
         disp
        recorder EnvelopeNode -file "Data/MaxForce$j.out" -node 1 -dof 1
```

```
        reaction

        analyze $nPts $dt

        remove recorders

    }

    set DispData [open "Data/MaxDisp1.out" r]
    set Disp1 [read $DispData]
    close $DispData
    set DispData [open "Data/MaxDisp2.out" r]
    set Disp2 [read $DispData]
    close $DispData
    set ForceData [open "Data/MaxForce1.out" r]
    set Force1 [read $ForceData]
    close $ForceData
    set ForceData [open "Data/MaxForce2.out" r]
    set Force2 [read $ForceData]
    close $ForceData
    set MaxD1 [lindex $Disp1 2]
    set MaxD2 [lindex $Disp2 2]
    set MaxF1 [lindex $Force1 2]
    set MaxF2 [lindex $Force2 2]

    set SpectraFile [open $outFile a+]
    puts $SpectraFile "$deltaY $MaxD1 $MaxF1 $MaxD2 $MaxF2"
    close $SpectraFile

}

puts "Done"
```

```
#   -----1-STORY BRACED FRAME MODEL-----

set L 192.0
set H 144.0

set E 29000

set Lbr [expr hypot($L,$H)]
set cos2Theta [expr ($L/$Lbr)**2]
set A [expr $K*$Lbr/$E/$cos2Theta]

model basic -ndm 2 -ndf 2

node 1   0   0
node 2   0  $H
node 3  $L  $H
node 4  $L   0

set myNode 3

mass $myNode $M 0

fix 1 1 1
fix 4 1 1

if {$alpha == 0} {
    set Mat 5
} elseif {$alpha == 1} {
    set littleE $E
    set bigE $E
    set Mat 1
} else {
    set littleE [expr $E*$alpha]
    set bigE [expr $E*(1-$alpha)]
    set Mat 4
}

uniaxialMaterial Elastic 1 $littleE
uniaxialMaterial ElasticPPGap 2 $bigE 1000000 [expr $deltaY/$Lbr]
uniaxialMaterial ElasticPPGap 3 $bigE -1000000 [expr -1*$deltaY/$Lbr]
uniaxialMaterial Parallel 4 1 2 3
uniaxialMaterial Steel01 5 50 $E 0.02

element Truss 1 1 2 1000000 1
element Truss 2 2 3 1000000 1
element Truss 3 3 4 1000000 1
element corotTruss 4 1 3 $A $Mat
```

```
#   -----SDOF-----

wipe all

set pi 3.14159
set g 386.4
set M 2.5
set zeta 0.02
set TMax 6
set TStep 0.05
set alphas "0.1 0.25 0.5 1.0"
set deltaYMin 5
set deltaYMax 30
set deltaYStep 5
set F [expr 1/2.54]

source ReadRecord2.tcl

for {set EQ 1} {$EQ <= 20} {incr EQ} {

    puts "-------------------- EQ = $EQ"

    set inFile "GroundMotions/SAC/LA$EQ.acc"
    set acceldata "GroundMotions/ATH.at2"
    ReadRecord $inFile $acceldata dt nPts

    for {set deltaY $deltaYMin} {$deltaY <= $deltaYMax} {incr deltaY
    $deltaYStep} {

        puts "---------deltaY = $deltaY"

        for {set a 0} {$a <= [expr [llength $alphas]-1]} {incr a} {

            set alpha [lindex $alphas $a]

            puts "alpha = $alpha"

            set a [expr $a+1]
            file mkdir "SpectraFiles/EQ$EQ/deltaY$deltaY"
            set outfile "SpectraFiles/EQ$EQ/deltaY$deltaY/spectra$a.out"
            file delete $outfile
            set a [expr $a-1]

            for {set i 1} {$i <= [expr $TMax/$TStep]} {incr i} {

                set T [expr $i*$TStep]
                set omega [expr 2*$pi/$T]
```

```
                set K [expr ($omega**2)*$M]

                wipe
                source 1StoryFrame.tcl
                source Analysis.tcl

                set SpectraFile [open $outfile a+]
                puts $SpectraFile "$T $MaxD $MaxA $MaxPA $MaxF"
                close $SpectraFile

            }

        }

    }

    puts "-------------"
    puts "Steel"

    set outfile "SpectraFiles/EQ$EQ/steel.out"
    file delete $outfile

    for {set i 1} {$i <= [expr $TMax/$TStep]} {incr i} {

        set T [expr $i*$TStep]
        set omega [expr 2*$pi/$T]
        set K [expr ($omega**2)*$M]

        wipe
        set alpha 0
        source 1StoryFrame.tcl
        source Analysis.tcl

        set SpectraFile [open $outfile a+]
        puts $SpectraFile "$T $MaxD $MaxA $MaxPA $MaxF"
        close $SpectraFile

    }

}

file mkdir "SpectraFiles"
set infofile [open "SpectraFiles/SpectraInfo.out" w]
puts  $infofile "$TMax $TStep $deltaYMin $deltaYMax $deltaYStep [llength $
alphas] $alphas"
close $infofile
```

```
#   -----READ RECORD-----

proc ReadRecord {inFilename outFilename dt nPts} {

    # Pass dt by reference
    upvar $dt DT
    upvar $nPts NPTS

    # Open the input file and catch the error if it can't be read
    if [catch {open $inFilename r} inFileID] {
    puts stderr "Cannot open $inFilename for reading"
    } else {

    set outFileID [open $outFilename w]

    set flag 0

    foreach line [split [read $inFileID] \n] {

        if {[llength $line] == 0} {
        # Blank line --> do nothing
        continue
        } elseif {$flag == 1 && $line != 0} {
        # Echo ground motion values to output file
        puts $outFileID $line
        } else {

        if {$flag == 0} {
         set flag 2
        } elseif {$flag == 2} {
            set NPTS [lindex $line 0]
            set DT [lindex $line 1]
            set flag 1
        }

        }
    }

    }


    # Close the output file
    close $outFileID

    # Close the input file
    close $inFileID
    }
```

```
#   -----ANALYSIS-----

recorder EnvelopeNode -file "Data/MaxDisp.out" -node $myNode -dof 1 disp
recorder EnvelopeNode -file "Data/MaxAccel.out" -node $myNode -dof 1 accel
recorder EnvelopeElement -file "Data/MaxForce.out" -ele $myEle axialForce

wipeAnalysis
constraints Plain
numberer Plain
system BandGeneral
test NormDispIncr 1.0e-8 10
algorithm Newton
integrator Newmark 0.5 0.25
analysis Transient

set accelSeries "Series -dt $dt -filePath $acceldata -factor $F"
pattern UniformExcitation 1 1 -accel $accelSeries
rayleigh 0. 0. 0. [expr 2*$zeta/$omega]

analyze $nPts $dt

remove recorders

set DispData [open "Data/MaxDisp.out" r]
set Disp [read $DispData]
close $DispData
set AccelData [open "Data/MaxAccel.out" r]
set Accel [read $AccelData]
close $AccelData
set ForceData [open "Data/MaxForce.out" r]
set Force [read $ForceData]
close $ForceData
set MaxD [lindex $Disp 2]
set MaxA [expr [lindex $Accel 2]/$g]
set MaxPA [expr $MaxD*($omega**2)/$g]
set MaxF [lindex $Force 2]
```

```
#   -----PUSHOVER ANALYSIS-----

wipe all

set dx 0.005
set nSteps 10000;       #Max for spreadsheet is 10000
set MaxDisp [expr $dx * $nSteps]
set Load 2;             #1=Single Point, 2=Seismic, 3=Wind

for {set i 3} {$i <= 3} {incr i} {
    set Mat [expr 10 * $i]
    puts "Building model $i of 3..."
    wipe
    source 12StoryFrame.tcl

    if {$Load == 1} {
        pattern Plain 1 Linear {
            load $myNode 1.0 0 0
        }
    } elseif {$Load == 2} {
        pattern Plain 1 Linear {
            load  3   0.8 0 0
            load  5   3.4 0 0
            load  7   7.6 0 0
            load  9  13.4 0 0
            load 11  21.0 0 0
            load 13  30.3 0 0
            load 15  41.2 0 0
            load 17  53.8 0 0
            load 19  68.1 0 0
            load 21  84.0 0 0
            load 23 101.7 0 0
            load 25 121.0 0 0
        }
    } elseif {$Load == 3} {
        pattern Plain 1 Linear {
            load  3  69.7 0 0
            load  5  80.5 0 0
            load  7  84.4 0 0
            load  9  90.6 0 0
            load 11  96.0 0 0
            load 13  97.5 0 0
            load 15 101.4 0 0
            load 17 105.3 0 0
            load 19 107.6 0 0
            load 21 110.7 0 0
            load 23 110.7 0 0
```

```
            load 25  56.5 0 0
        }
    }

    constraints Plain
    numberer RCM
    system BandGeneral
    test NormUnbalance 1.0e-6 400
    algorithm Newton
    integrator DisplacementControl $myNode 1 $dx
    analysis Static

    recorder Node -file "PushoverData/Force$i.out" -node 1 -dof 1 reaction
    recorder Node -file "PushoverData/Disp.out" -node $myNode -dof 1 disp

    puts "Performing pushover analysis..."

    set currentDisp 0.0
    set ok 0
    while {$ok == 0 && $currentDisp < $MaxDisp} {

        set ok [analyze 1]

        # if the analysis fails try initial tangent iteration
        if {$ok != 0} {
            puts "regular newton failed .. lets try an initail stiffness for
            this step"
            test NormDispIncr 1.0e-12  1000
            algorithm ModifiedNewton -initial
            set ok [analyze 1]
            if {$ok == 0} {puts "that worked .. back to regular newton"}
            test NormDispIncr 1.0e-12  10
            algorithm Newton
        }

        set currentDisp [nodeDisp $myNode 1]
    }


    if {$ok == 0} {
      puts "Pushover analysis completed SUCCESSFULLY";
    } else {
      puts "Pushover analysis FAILED";
    }
    remove recorders
}
```

```
#   ----FIND PERIOD----

set nMode 5
set pi 3.14159265

set PerFile [open "PeriodData.out" w]

for {set i 1} {$i <=3} {incr i} {
    set Mat [expr $i*10]
    wipe
    source 12StoryFrame.tcl
    puts "Material $i:"
    puts $PerFile "Material $i:"
    set w [eigen $nMode]
    for {set j 1} {$j <= $nMode} {incr j} {
        set T [expr 2 * $pi / ([lindex $w [expr $j-1]]**0.5)]
        puts "Mode $j: = $T"
        puts $PerFile "Mode $j: = $T"
    }
}

close $PerFile
```

```
#   ----MDOF ANALYSIS----

wipe all

set Mat 30
source 12StoryFrame.tcl
source DynamicAnalysis.tcl
puts "Done"
```

```
#   ----12-STORY BRACED FRAME MODEL----

puts "Building the model..."

set L 393.7
set H 196.9
set D 5.8e-4
set g 386.4
set deltaY 0.447
set ColBeamMat 10
set ColBeamDef 1
set PdeltaCol 1
set alphas "0.67 0.63 0.58 0.53 0.50 0.44 0.41 0.36 0.34 0.30 0.23 0.14"
set deltaYs "0.0035 0.0028 0.0023 0.0022 0.0023 0.0023 0.0024 0.0024 0.0026
0.0027 0.0025 0.0024"
set Es "28221 26451 24406 22262 19961 19275 16549 15239 13476 11095 9345 5264"
set EScale 1
set EScales "1.00 1.00 1.00 1.19 1.41 1.63 1.88 2.14 2.45 2.74 3.00 3.23"
set E 29000

model basic -ndm 2 -ndf 3

node  1  0   0
node  2  $L  0
node  3  0   [expr 1*$H]
node  4  $L  [expr 1*$H]
node  5  0   [expr 2*$H]
node  6  $L  [expr 2*$H]
node  7  0   [expr 3*$H]
node  8  $L  [expr 3*$H]
node  9  0   [expr 4*$H]
node 10  $L  [expr 4*$H]
node 11  0   [expr 5*$H]
node 12  $L  [expr 5*$H]
node 13  0   [expr 6*$H]
node 14  $L  [expr 6*$H]
node 15  0   [expr 7*$H]
node 16  $L  [expr 7*$H]
node 17  0   [expr 8*$H]
node 18  $L  [expr 8*$H]
node 19  0   [expr 9*$H]
node 20  $L  [expr 9*$H]
node 21  0   [expr 10*$H]
node 22  $L  [expr 10*$H]
node 23  0   [expr 11*$H]
node 24  $L  [expr 11*$H]
node 25  0   [expr 12*$H]
```

```
node 26  $L  [expr 12*$H]

fix 1 1 1 0
fix 2 1 1 0

uniaxialMaterial Elastic 10 $E
uniaxialMaterial Steel01 20 50 $E 0.02

if {$Mat == 10} {
    for {set i 101} {$i <= 112} {incr i} {
        uniaxialMaterial Elastic $i $E
    }
} elseif {$Mat == 20} {
    for {set i 101} {$i <= 112} {incr i} {
        uniaxialMaterial Steel01 $i 50 $E 0.02
    }
} elseif {$Mat == 30} {
    for {set i 101} {$i <= 112} {incr i} {
        set E [lindex $Es [expr $i-101]]
        #set E [expr $E*(1+($EScale-1)*($i-100)/12)]
        set EScale [lindex $EScales [expr $i-101]]
        set E [expr $E*$EScale]
        set alpha [lindex $alphas [expr $i-101]]
        set alpha 0.1
        set deltaY [lindex $deltaYs [expr $i-101]]
        set deltaY 0.0038
        uniaxialMaterial ElasticPPGap [expr $i+200] [expr $E/$alpha-$E]
        100000 $deltaY
        uniaxialMaterial ElasticPPGap [expr $i+300] [expr $E/$alpha-$E]
        -100000 [expr -1 * $deltaY]
        uniaxialMaterial Elastic [expr $i+400] $E
        uniaxialMaterial Parallel $i [expr $i+200] [expr $i+300] [expr $i+400]
    }
}

set i [expr $Mat/10]

set A11 134.0
set A12 109.0
set A13  83.3
set A14  56.8
set A15  32.0
set A16  17.9
set A21  25.3
set A22  22.6

set I11 7190
```

```
set I12 5440
set I13 3840
set I14 2400
set I15 1240
set I16  640

source MatIndex.tcl
puts "Beam and column material: [MatIndex $ColBeamMat]"
puts "Brace material: [MatIndex $Mat]"

if {$ColBeamDef == 0} {
    puts "Beam and column deformation *not* considered"
    set A11 10000000
    set A12 10000000
    set A13 10000000
    set A14 10000000
    set A15 10000000
    set A16 10000000
    set A21 10000000
    set A22 10000000
} else {
    puts "Beam and column deformation considered"
}

geomTransf Linear 1
element elasticBeamColumn 101  1  3 $A11 $E $I11 1
element elasticBeamColumn 102  2  4 $A11 $E $I11 1
element elasticBeamColumn 103  3  5 $A11 $E $I11 1
element elasticBeamColumn 104  4  6 $A11 $E $I11 1
element elasticBeamColumn 105  5  7 $A12 $E $I12 1
element elasticBeamColumn 106  6  8 $A12 $E $I12 1
element elasticBeamColumn 107  7  9 $A12 $E $I12 1
element elasticBeamColumn 108  8 10 $A12 $E $I12 1
element elasticBeamColumn 109  9 11 $A13 $E $I13 1
element elasticBeamColumn 110 10 12 $A13 $E $I13 1
element elasticBeamColumn 111 11 13 $A13 $E $I13 1
element elasticBeamColumn 112 12 14 $A13 $E $I13 1
element elasticBeamColumn 113 13 15 $A14 $E $I14 1
element elasticBeamColumn 114 14 16 $A14 $E $I14 1
element elasticBeamColumn 115 15 17 $A14 $E $I14 1
element elasticBeamColumn 116 16 18 $A14 $E $I14 1
element elasticBeamColumn 117 17 19 $A15 $E $I15 1
element elasticBeamColumn 118 18 20 $A15 $E $I15 1
element elasticBeamColumn 119 19 21 $A15 $E $I15 1
element elasticBeamColumn 120 20 22 $A15 $E $I15 1
element elasticBeamColumn 121 21 23 $A16 $E $I16 1
element elasticBeamColumn 122 22 24 $A16 $E $I16 1
```

```
element elasticBeamColumn 123 23 25 $A16 $E $I16 1
element elasticBeamColumn 124 24 26 $A16 $E $I16 1
element corotTruss 201  3  4 $A21 $ColBeamMat
element corotTruss 202  5  6 $A21 $ColBeamMat
element corotTruss 203  7  8 $A21 $ColBeamMat
element corotTruss 204  9 10 $A21 $ColBeamMat
element corotTruss 205 11 12 $A21 $ColBeamMat
element corotTruss 206 13 14 $A21 $ColBeamMat
element corotTruss 207 15 16 $A21 $ColBeamMat
element corotTruss 208 17 18 $A21 $ColBeamMat
element corotTruss 209 19 20 $A21 $ColBeamMat
element corotTruss 210 21 22 $A21 $ColBeamMat
element corotTruss 211 23 24 $A21 $ColBeamMat
element corotTruss 212 25 26 $A22 $ColBeamMat
element corotTruss 301  1  4 11.0 101
element corotTruss 302  3  6 11.0 102
element corotTruss 303  5  8 11.0 103
element corotTruss 304  7 10 11.0 104
element corotTruss 305  9 12 11.0 105
element corotTruss 306 11 14 10.0 106
element corotTruss 307 13 16 10.0 107
element corotTruss 308 15 18  9.0 108
element corotTruss 309 17 20  8.0 109
element corotTruss 310 19 22  7.0 110
element corotTruss 311 21 24  5.0 111
element corotTruss 312 23 26  3.0 112

set M [expr (5*$L)*(5*$L)*$D/$g/4]
set M1 [expr (1.5*$L)*(0.5*$L)*$D/$g]
set M2 [expr $M-$M1]

if {$PdeltaCol == 1} {
    puts "P-delta column provided"

    node 401 [expr 2*$L] 0
    node 402 [expr 2*$L] [expr 1*$H]
    node 403 [expr 2*$L] [expr 2*$H]
    node 404 [expr 2*$L] [expr 3*$H]
    node 405 [expr 2*$L] [expr 4*$H]
    node 406 [expr 2*$L] [expr 5*$H]
    node 407 [expr 2*$L] [expr 6*$H]
    node 408 [expr 2*$L] [expr 7*$H]
    node 409 [expr 2*$L] [expr 8*$H]
    node 410 [expr 2*$L] [expr 9*$H]
    node 411 [expr 2*$L] [expr 10*$H]
    node 412 [expr 2*$L] [expr 11*$H]
    node 413 [expr 2*$L] [expr 12*$H]
```

```
mass  4 $M1 0 0
mass  6 $M1 0 0
mass  8 $M1 0 0
mass 10 $M1 0 0
mass 12 $M1 0 0
mass 14 $M1 0 0
mass 16 $M1 0 0
mass 18 $M1 0 0
mass 20 $M1 0 0
mass 22 $M1 0 0
mass 24 $M1 0 0
mass 26 $M1 0 0

mass 402 $M2 0 0
mass 403 $M2 0 0
mass 404 $M2 0 0
mass 405 $M2 0 0
mass 406 $M2 0 0
mass 407 $M2 0 0
mass 408 $M2 0 0
mass 409 $M2 0 0
mass 410 $M2 0 0
mass 411 $M2 0 0
mass 412 $M2 0 0
mass 413 $M2 0 0

fix 401 1 1 0
equalDOF  4 402 1
equalDOF  6 403 1
equalDOF  8 404 1
equalDOF 10 405 1
equalDOF 12 406 1
equalDOF 14 407 1
equalDOF 16 408 1
equalDOF 18 409 1
equalDOF 20 410 1
equalDOF 22 411 1
equalDOF 24 412 1
equalDOF 26 413 1

set A51 284.6
set A52 248.4
set A53 198.5
set A54 149.2
set A55 120.2
set A56  87.5
```

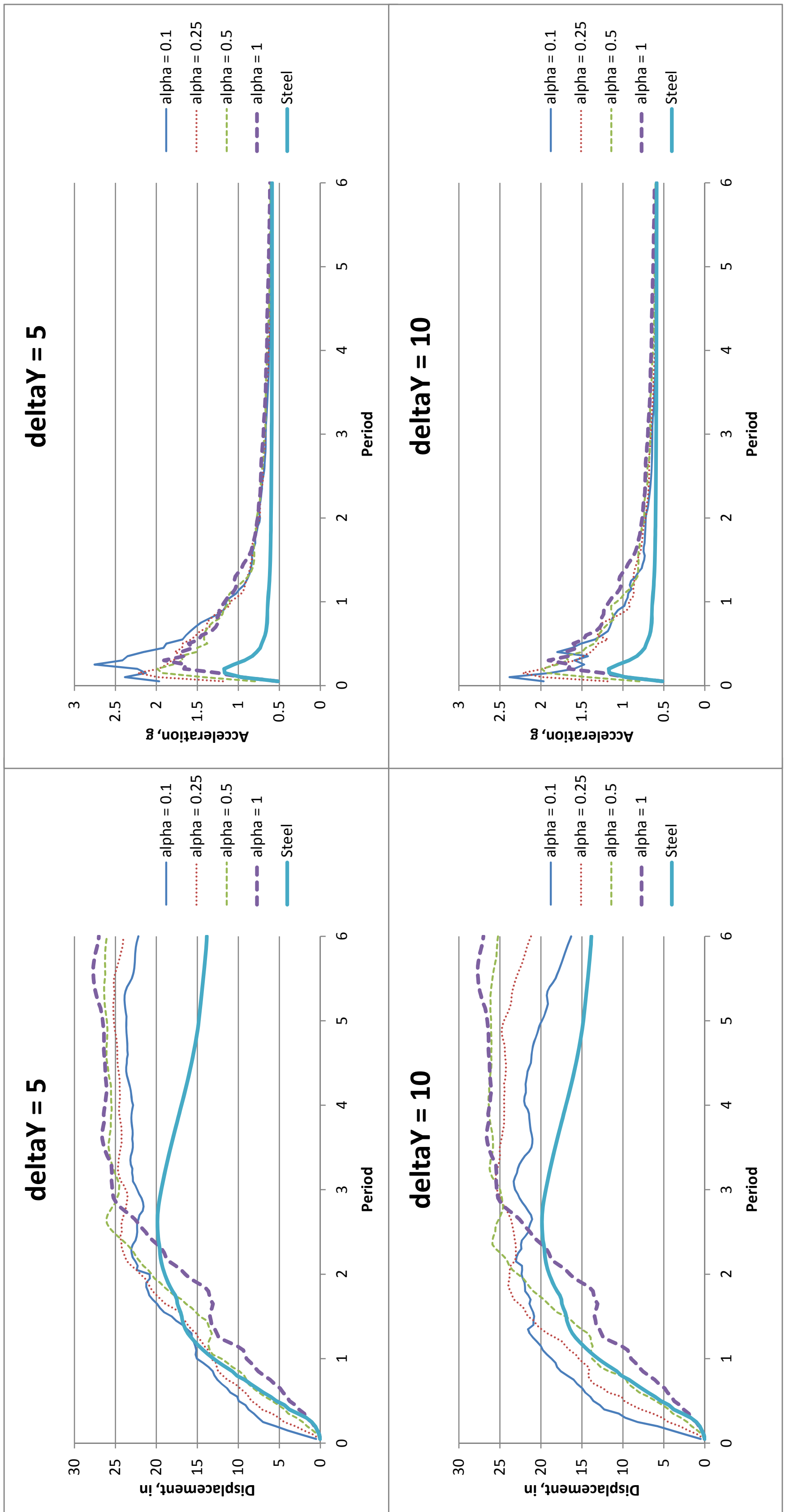
```
    set I51 9099
    set I52 7706
    set I53 5879
    set I54 4235
    set I55 3305
    set I56 2313

    element elasticBeamColumn 501 401 402 $A51 $E $I51 1
    element elasticBeamColumn 502 402 403 $A51 $E $I51 1
    element elasticBeamColumn 503 403 404 $A52 $E $I52 1
    element elasticBeamColumn 504 404 405 $A52 $E $I52 1
    element elasticBeamColumn 505 405 406 $A53 $E $I53 1
    element elasticBeamColumn 506 406 407 $A53 $E $I53 1
    element elasticBeamColumn 507 407 408 $A54 $E $I54 1
    element elasticBeamColumn 508 408 409 $A54 $E $I54 1
    element elasticBeamColumn 509 409 410 $A55 $E $I55 1
    element elasticBeamColumn 510 410 411 $A55 $E $I55 1
    element elasticBeamColumn 511 411 412 $A56 $E $I56 1
    element elasticBeamColumn 512 412 413 $A56 $E $I56 1
} else {
    puts "P-delta column *not* provided"

    mass  4 $M 0 0
    mass  6 $M 0 0
    mass  8 $M 0 0
    mass 10 $M 0 0
    mass 12 $M 0 0
    mass 14 $M 0 0
    mass 16 $M 0 0
    mass 18 $M 0 0
    mass 20 $M 0 0
    mass 22 $M 0 0
    mass 24 $M 0 0
    mass 26 $M 0 0
}

set myNode 26
```

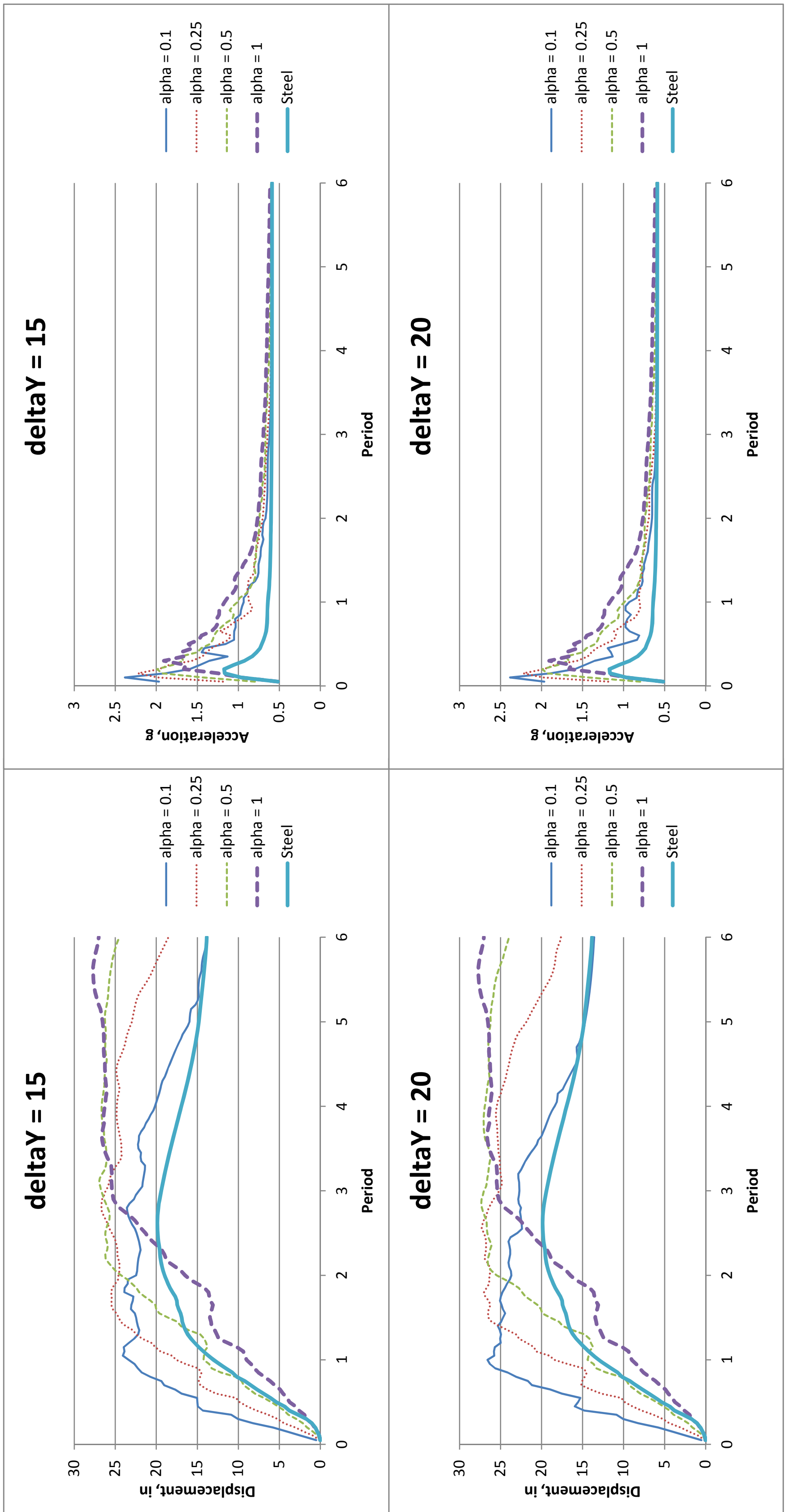
```
#   -----DYNAMIC ANALYSIS-----

source ReadRecord.tcl
source getATH.tcl

puts "Analyzing model under 20 EQ time histories..."

source readrecord2.tcl

for {set i 1} {$i <= 20} {incr i} {

    reset

    puts "EQ = $i"

    set inFile "GroundMotions/SAC/LA$i.acc"
    set acceldata "GroundMotions/ATH.at2"
    ReadRecord $inFile $acceldata dt nPts
    set F [expr 1/2.54]

    for {set j 1} {$j <= 12} {incr j} {
        recorder Drift -file "Data/Drift$j.out" -iNode [expr 2*$j] -jNode [
        expr 2*($j+1)] -dof 1 -perpDirn 2
    }
    recorder EnvelopeNode -file "Data/Disp.out" -node $myNode -dof 1 disp
    recorder EnvelopeNode -file "Data/Accel.out" -node $myNode -dof 1 accel
    recorder EnvelopeNode -file "Data/BaseShear.out" -node 1 -dof 1 reaction

    wipeAnalysis

    constraints Plain
    numberer Plain
    system BandGeneral
    test NormDispIncr 1.0e-8 10
    algorithm Newton
    integrator Newmark 0.5 0.25
    analysis Transient

    set accelSeries "Series -dt $dt -filePath $acceldata -factor $F"
    pattern UniformExcitation $i 1 -accel $accelSeries
    rayleigh 0. 0. 0. [expr 2*0.02/([eigen 1]**0.5)]

    analyze [expr round($nPts*12.0)] $dt

    remove recorders

    set outfile "Data/EQ$i.out"
```

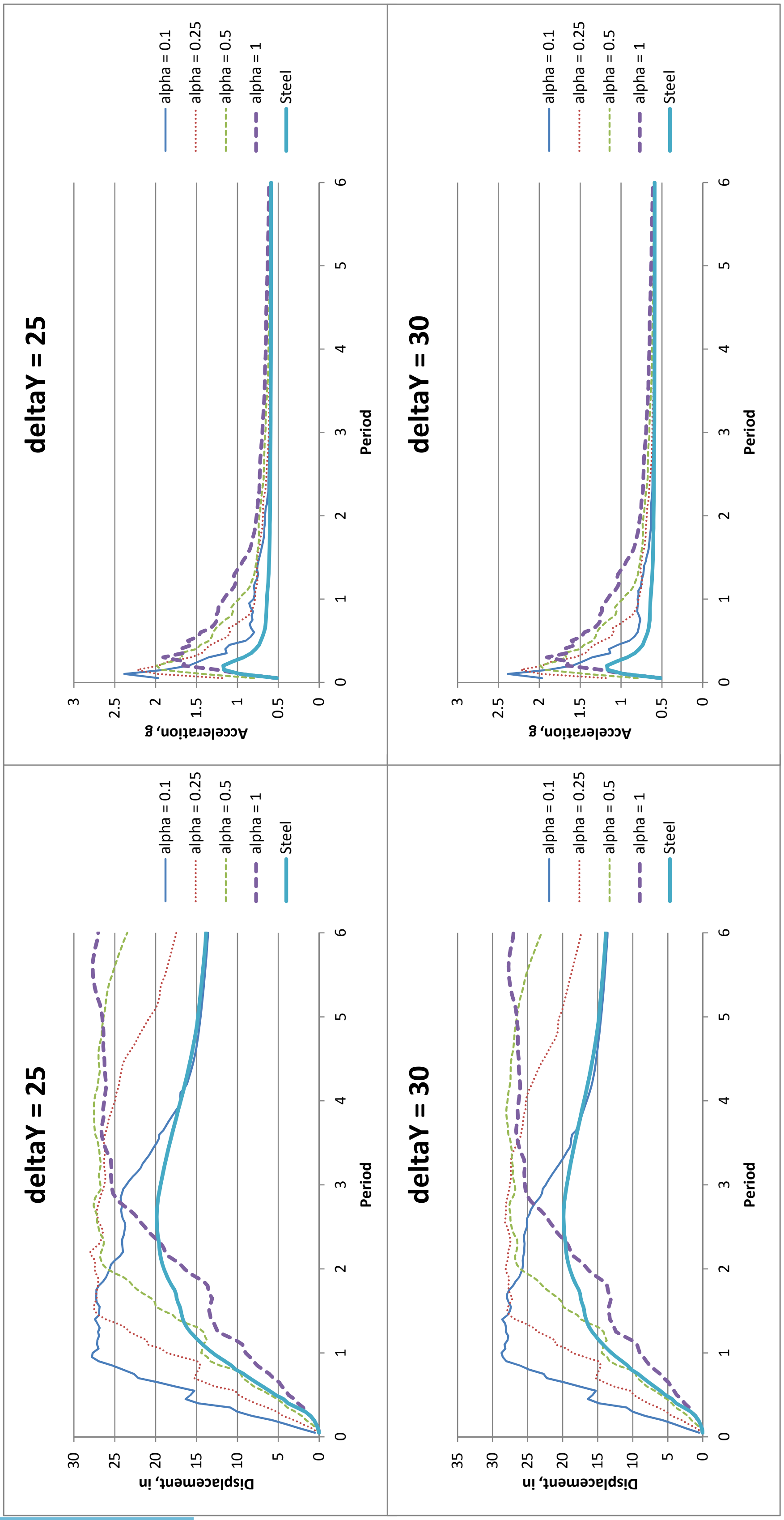
```
    file delete $outfile
    set DataFile [open $outfile a+]
    for {set j 1} {$j <= 12} {incr j} {
        set DriftFile [open "Data/Drift$j.out" r]
        set MaxDrift 0
        foreach line [split [read $DriftFile] \n] {
            if {$line == ""} {
                continue
            } else {
                set Drift [expr abs([lindex $line 0])]
                if {$Drift > $MaxDrift} {
                    set MaxDrift $Drift
                }
            }
        }
        close $DriftFile
        puts $DataFile "$MaxDrift $Drift"
    }
    set DFile [open "Data/Disp.out" r]
    set DData [read $DFile]
    set DData [lindex $DData 2]
    set AFile [open "Data/Accel.out" r]
    set AData [read $AFile]
    set AData [lindex $AData 2]
    set VFile [open "Data/BaseShear.out" r]
    set VData [read $VFile]
    set VData [lindex $VData 2]
    puts $DataFile $DData
    puts $DataFile $AData
    puts $DataFile $VData
    close $DataFile

}
```

APPENDIX B: SDOF PLOTS

Found in this appendix are the plots from the single-degree-of-freedom parameter study.


deltaY = 5
Displacement, in
Acceleration, g
Period
deltaY = 10
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel


deltaY = 15
Displacement, in
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel
deltaY = 15
Acceleration, g
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel
deltaY = 20
Displacement, in
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel
deltaY = 20
Acceleration, g
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel



deltaY = 25
Displacement, in
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel
deltaY = 25
Acceleration, g
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel
deltaY = 30
Displacement, in
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel
deltaY = 30
Acceleration, g
Period
alpha = 0.1
alpha = 0.25
alpha = 0.5
alpha = 1
Steel

APPENDIX C: STRUCTURAL CALCULATIONS

Found in this appendix are the calculations for the structural design of the building used in the case study.

<u>Building data:</u>

Dimensions:
(5) bays x (5) bays x (12) stories
Story height, $h = 5m = 196.9 \cdot in$
Bay width, $w = 10m = 393.7 \cdot in$

Loading:

$$D = 4kPa = 5.8 \times 10^{-4} \cdot ksi$$
$$L = 2kPa$$
$$L_r = 1kPa$$

<u>Gravity Design</u>

Roof Beam:

$$l = w = 32.8 \cdot ft$$
$$s = \frac{l}{3} = 10.9 \cdot ft$$
$$A_t = l \cdot s = 359 \cdot ft^2$$
$$L'_r = \max\left(\min\left(1.2 - 0.001 \cdot A_t \cdot ft^{-2}, 1.0\right), 0.6\right) \cdot L_r = 17.6\,psf$$
$$w_u = \left(1.2 \cdot D + 1.6 \cdot L'_r\right) \cdot s = 1.4 \cdot \frac{kip}{ft}$$
$$M_u = \frac{w_u \cdot l^2}{8} = 189 \cdot kip \cdot ft \qquad L_b = 0$$
$$V_u = \frac{w_u \cdot l}{2} = 23\,kip$$
$$\delta_{max} = \frac{l}{360} = 1.09 \cdot in \qquad I_{min} = \frac{5 \cdot L'_r \cdot s \cdot l^4}{384 \cdot E \cdot \delta_{max}} = 158 \cdot in^4$$

Use a W12x35

Roof Girder:

$$A_t = l \cdot l = 1076 \cdot ft^2$$
$$L'_r = \max\left(\min\left(1.2 - 0.001 \cdot A_t \cdot ft^{-2}, 1.0\right), 0.6\right) \cdot L_r = 12.5\,psf$$
$$w_u = \left(1.2 \cdot D + 1.6 \cdot L'_r\right) \cdot s = 1.3 \cdot \frac{kip}{ft}$$
$$P_u = w_u \cdot l = 43.2\,kip$$
$$M_u = P_u \cdot \frac{l}{3} = 472 \cdot kip \cdot ft \qquad L_b = \frac{l}{3} = 10.9 \cdot ft$$
$$V_u = P_u = 43.2\,kip$$
$$I_{min} = \frac{\left(L'_r \cdot s \cdot \frac{l}{2}\right) \cdot l^3}{28 \cdot E \cdot \delta_{max}} = 154 \cdot in^4$$

Use a W16x77

Floor Beam:

$$A_t = l \cdot s = 359 \cdot ft^2$$

$$L' = \max\left(\min\left(0.25 + \frac{15}{\sqrt{2 \cdot A_t \cdot ft^{-2}}}, 1.0\right), 0.5\right) \cdot L = 33.8\, psf$$

$$w_u = (1.2 \cdot D + 1.6 \cdot L') \cdot s = 1.7 \cdot \frac{kip}{ft}$$

$$M_u = \frac{w_u \cdot l^2}{8} = 227 \cdot kip \cdot ft \qquad L_b = 0$$

$$V_u = \frac{w_u \cdot l}{2} = 27.7\, kip$$

$$\delta_{max} = \frac{l}{360} = 1.09 \cdot in$$

$$I_{min} = \frac{5 \cdot L' \cdot s \cdot l^4}{384 \cdot E \cdot \delta_{max}} = 304 \cdot in^4$$

Use a W16x36

Floor Girder:

$$A_t = l \cdot l = 1076 \cdot ft^2$$

$$L' = \max\left(\min\left(0.25 + \frac{15}{\sqrt{2 \cdot A_t \cdot ft^{-2}}}, 1.0\right), 0.5\right) \cdot L = 23.9\, psf$$

$$w_u = (1.2 \cdot D + 1.6 \cdot L') \cdot s = 1.5 \cdot \frac{kip}{ft}$$

$$P_u = w_u \cdot l = 49.7\, kip$$

$$M_u = P_u \cdot \frac{l}{3} = 544 \cdot kip \cdot ft \qquad L_b = \frac{l}{3} = 10.9 \cdot ft$$

$$V_u = P_u = 49.7\, kip$$

$$I_{min} = \frac{\left(L' \cdot s \cdot \frac{l}{2}\right) \cdot l^3}{28 \cdot E \cdot \delta_{max}} = 295 \cdot in^4$$

Use a W18x86

Corner columns:

$$A_t = \frac{1}{2}\cdot\frac{1}{2} = 269.1\cdot ft^2$$

$$L'_r = \max\left(\min\left(1.2 - 0.001\cdot A_t\cdot ft^{-2}, 1.0\right), 0.6\right)\cdot L_r = 19.4\,psf$$

$$L' = \max\left(\min\left(0.25 + \frac{15}{\sqrt{4\cdot A_t\cdot ft^{-2}}}, 1.0\right), 0.4\right)\cdot L = 29.5\,psf$$

$$P_u = 1.2\cdot D\cdot 2n\,A_t + 0.5\cdot L'_r\cdot A_t + 1.6L'\cdot A_t\cdot(2n-1) = \begin{pmatrix} 69 \\ 149 \\ 228 \\ 307 \\ 387 \\ 466 \end{pmatrix} kip \qquad \text{Use} \begin{pmatrix} W12x40 \\ W12x40 \\ W12x40 \\ W12x50 \\ W12x53 \\ W12x58 \end{pmatrix}$$

Edge columns:

$$A_t = l\cdot\frac{1}{2} = 538.2\cdot ft^2$$

$$L'_r = \max\left(\min\left(1.2 - 0.001\cdot A_t\cdot ft^{-2}, 1.0\right), 0.6\right)\cdot L_r = 13.8\,psf$$

$$L' = \max\left(\min\left(0.25 + \frac{15}{\sqrt{4\cdot A_t\cdot ft^{-2}}}, 1.0\right), 0.4\right)\cdot L = 23.9\,psf$$

$$P_u = 1.2\cdot D\cdot 2n\,A_t + 0.5\cdot L'_r\cdot A_t + 1.6L'\cdot A_t\cdot(2n-1) = \begin{pmatrix} 132 \\ 281 \\ 431 \\ 580 \\ 729 \\ 878 \end{pmatrix} kip \qquad \text{Use} \begin{pmatrix} W12x40 \\ W12x50 \\ W12x53 \\ W12x65 \\ W12x79 \\ W12x96 \end{pmatrix}$$

Center columns:

$$A_t = l\cdot l = 1076.4\cdot ft^2$$

$$L'_r = \max\left(\min\left(1.2 - 0.001\cdot A_t\cdot ft^{-2}, 1.0\right), 0.6\right)\cdot L_r = 12.5\,psf$$

$$L' = \max\left(\min\left(0.25 + \frac{15}{\sqrt{4\cdot A_t\cdot ft^{-2}}}, 1.0\right), 0.4\right)\cdot L = 20\,psf$$

$$P_u = 1.2\cdot D\cdot 2n\,A_t + 0.5\cdot L'_r\cdot A_t + 1.6L'\cdot A_t\cdot(2n-1) = \begin{pmatrix} 257 \\ 542 \\ 826 \\ 1111 \\ 1396 \\ 1680 \end{pmatrix} kip \qquad \text{Use} \begin{pmatrix} W12x45 \\ W12x65 \\ W12x87 \\ W12x120 \\ W12x152 \\ W12x170 \end{pmatrix}$$

Lateral Design

Seismic Loads:

$A = (51)\cdot(51) = 26910\cdot ft^2$

$W = 12\cdot\frac{A}{4}\cdot D = 6744.3\ kip$

$C_t = 0.03$

$h_n = \frac{12\cdot h}{ft}$

$x = 0.75$

$T_a = C_t\cdot h_n{}^x\cdot sec = 1.6\,s$

$SA = 0.62g$

$R = 8 \qquad I = 1.0$

$$C_s = \frac{\frac{SA}{g}}{\left(\frac{R}{I}\right)} = 0.078$$

$V = C_s\cdot W = 522.7\ kip$

$w_x = \frac{A\cdot D\cdot n}{kip} \qquad h_x = \frac{h\cdot s}{ft} \qquad k = 2$

$$C_{vx} = \frac{\overrightarrow{\left(w_x\cdot h_x{}^k\right)}}{\sum w_x\cdot h_x{}^k} = \begin{pmatrix} 0.222 \\ 0.186 \\ 0.154 \\ 0.125 \\ 0.098 \\ 0.075 \\ 0.055 \\ 0.038 \\ 0.025 \\ 0.014 \\ 0.006 \\ 0.002 \end{pmatrix} \qquad F_x = C_{vx}\cdot V = \begin{pmatrix} 115.8 \\ 97.3 \\ 80.4 \\ 65.1 \\ 51.5 \\ 39.4 \\ 28.9 \\ 20.1 \\ 12.9 \\ 7.2 \\ 3.2 \\ 0.8 \end{pmatrix} kip$$

BRB Brace Design:

$$L_{br} = \sqrt{w^2 + h^2} = 36.7\,ft$$

$$F_{ybrb} = 60ksi$$

$$x = 1 .. 12$$

$$F_t(x) = \sum submatrix\left(F_x, 0, x-1, 0, 0\right) \qquad F_{brb}(x) = F_t(x)\cdot\frac{L_{br}}{w} \qquad A_{brb}(x) = Ceil\left(\frac{F_{brb}(x)}{0.9\cdot F_{ybrb}}, 1in^2\right)$$

$$F_t(x) = \begin{pmatrix} 115.8 \\ 213.1 \\ 293.5 \\ 358.6 \\ 410.1 \\ 449.5 \\ 478.5 \\ 498.6 \\ 511.4 \\ 518.7 \\ 521.9 \\ 522.7 \end{pmatrix} kip \qquad F_{brb}(x) = \begin{pmatrix} 129.5 \\ 238.2 \\ 328.1 \\ 401 \\ 458.5 \\ 502.6 \\ 534.9 \\ 557.4 \\ 571.8 \\ 579.9 \\ 583.5 \\ 584.4 \end{pmatrix} kip \qquad A_{brb}(x) = \begin{pmatrix} 3 \\ 5 \\ 7 \\ 8 \\ 9 \\ 10 \\ 10 \\ 11 \\ 11 \\ 11 \\ 11 \\ 11 \end{pmatrix} \cdot in^2$$

BRBF Column Design:

$$F_{cap}(x) = A_{brb}(x)\cdot F_{ybrb}\cdot 1.8$$

$$F_{cap}(x) = \begin{pmatrix} 324 \\ 540 \\ 756 \\ 864 \\ 972 \\ 1080 \\ 1080 \\ 1188 \\ 1188 \\ 1188 \\ 1188 \\ 1188 \end{pmatrix} kip$$

Wind load calculations

$B := 5\cdot(10m) = 164.042ft$ $L := B = 164.042ft$ $h := 12\cdot(5m) = 196.85ft$ $n_1 := 12.7$

$V := 115mph$ $K_d := 0.85$ $Exp := C$

$z' := 0.6\cdot\frac{h}{ft} = 118.11$ $c := 0.20$ $\beta := 0.05$

$$I_{z'} := c\cdot\left(\frac{33}{z'}\right)^{\frac{1}{6}} = 0.162$$

$l := 500ft$ $\varepsilon' := \frac{1}{5.0}$

$$L_{z'} := l\cdot\left(\frac{z'}{33}\right)^{\varepsilon'} = 196.671m$$

$$Q := \sqrt{\frac{1}{1 + 0.63\cdot\left(\frac{B+h}{L_{z'}}\right)^{0.63}}} = 0.834$$

$g_Q := 3.4$ $g_v := 3.4$ $g_R := \sqrt{2\cdot\ln(3600n_1)} + \frac{0.577}{\sqrt{2\cdot\ln(3600n_1)}} = 4.757$

$\alpha' := \frac{1}{6.5}$ $b' := 0.65$ $V'_{z'} := b'\cdot\left(\frac{z'}{33}\right)^{\alpha'}\cdot\left(\frac{88}{60}\right)\cdot V = 133.395mph$

$\eta_h := 4.6\cdot\frac{n_1}{s}\cdot\frac{h}{V'_{z'}} = 58.78$ $\eta_B := 4.6\cdot\frac{n_1}{s}\cdot\frac{B}{V'_{z'}} = 48.983$ $\eta_L := 4.6\cdot\frac{n_1}{s}\cdot\frac{L}{V'_{z'}} = 48.983$

$$R_h := \frac{1}{\eta_h} - \frac{1}{2\cdot\eta_h^{2}}\left(1 - e^{-2\cdot\eta_h}\right) = 0.017$$

$$R_B := \frac{1}{\eta_B} - \frac{1}{2\cdot\eta_B^{2}}\left(1 - e^{-2\cdot\eta_B}\right) = 0.02$$

$$R_L := \frac{1}{\eta_L} - \frac{1}{2\cdot\eta_L^{2}}\left(1 - e^{-2\cdot\eta_L}\right) = 0.02$$

$N_1 := \frac{n_1\cdot L_{z'}}{s\cdot V'_{z'}} = 41.885$ $R_n := \frac{7.47N_1}{\left(1 + 10.3N_1\right)^{\frac{5}{3}}} = 0.013$

$$R := \sqrt{\frac{1}{\beta}\cdot R_n\cdot R_h\cdot R_B\cdot\left(0.53 + 0.47R_L\right)} = 6.822\times 10^{-3}$$

$G_f := 0.925\left(\frac{1 + 1.7\cdot I_{z'}\sqrt{g_Q^{2}\cdot Q^{2} + g_R^{2}\cdot R^{2}}}{1 + 1.7\cdot g_v\cdot I_{z'}}\right) = 0.851$ $GC_{pi} := 0.18$

$K_h := 1.46$

$$q_h := 0.00256 \cdot K_h \cdot K_d \cdot \left(\frac{V}{mph}\right)^2 \cdot psf = 42 \cdot psf$$

$$z := 5m \cdot n = \begin{pmatrix} 60 \\ 55 \\ 50 \\ 45 \\ 40 \\ 35 \\ 30 \\ 25 \\ 20 \\ 15 \\ 10 \\ 5 \end{pmatrix} m \qquad K_z := \begin{pmatrix} 1.46 \\ 1.43 \\ 1.43 \\ 1.39 \\ 1.36 \\ 1.31 \\ 1.26 \\ 1.24 \\ 1.17 \\ 1.09 \\ 1.04 \\ 0.90 \end{pmatrix} \qquad q_z := 0.00256 \cdot K_z \cdot K_d \cdot \left(\frac{V}{mph}\right)^2 \cdot psf = \begin{pmatrix} 42 \\ 41.2 \\ 41.2 \\ 40 \\ 39.1 \\ 37.7 \\ 36.3 \\ 35.7 \\ 33.7 \\ 31.4 \\ 29.9 \\ 25.9 \end{pmatrix} \cdot psf$$

$$h_{trib} := \begin{pmatrix} 2.5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \end{pmatrix} m \qquad F_w := \overrightarrow{\left(h_{trib} \cdot B \cdot q_z\right)} = \begin{pmatrix} 56.5 \\ 110.7 \\ 110.7 \\ 107.6 \\ 105.3 \\ 101.4 \\ 97.6 \\ 96 \\ 90.6 \\ 84.4 \\ 80.5 \\ 69.7 \end{pmatrix} \cdot kip$$

Alpha (α) Calculations

V	115	mph	b	10	m
V	51.4096	m/s	h	5	m
K_d	0.85		L_{br}	11.18	m
Story height	5	m	L_{br}	440.2	in
Drift Limit	L/400		θ	0.46	deg
Δ	0.0125	m			
Δ	0.4921	in			

Story	z [m]	K_z	q_z [kPa]	h_{trib} [m]	F_{story} [kN]	F_w [kN]
12	60	1.46	2.010577	2.5	251	251
11	55	1.43	1.9692638	5	492	744
10	50	1.43	1.9692638	5	492	1236
9	45	1.39	1.9141795	5	479	1714
8	40	1.36	1.8728662	5	468	2183
7	35	1.31	1.8040109	5	451	2634
6	30	1.26	1.7351555	5	434	3068
5	25	1.24	1.7076133	5	427	3494
4	20	1.17	1.6112158	5	403	3897
3	15	1.09	1.5010472	5	375	4272
2	10	1.04	1.4321918	5	358	4631
1	5	0.9	1.2393968	5	310	4940

Story	F_w [kip]	F_{brace} [kip]	A_{brace} [in^2]	$k_{req'd}$ [kip/in]	$E_{req'd}$ [ksi]
12	56.5	14.1	3	28.70	5264
11	167.2	41.8	5	84.93	9345
10	277.9	69.5	7	141.15	11095
9	385.4	96.4	8	195.80	13467
8	490.7	122.7	9	249.27	15239
7	592.1	148.0	10	300.78	16549
6	689.6	172.4	10	350.32	19275
5	785.6	196.4	11	399.07	19961
4	876.1	219.0	11	445.07	22262
3	960.5	240.1	11	487.93	24406
2	1041.0	260.2	11	528.82	26451
1	1110.6	277.7	11	564.21	28221